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USE OF AN EXTENDED KALMAN FILTER

by

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The error introduced by the DF accuracy of three Direction Finding equipments used on board two surface units providing bearing measurements, is used to evaluate the performance of the ESM systems using the Kalman Filter equations. The estimation process is evaluated as the error introduced in the DF accuracy of the ESM system is increased. The process is conducted with three different maneuvers involving a change in course of 0, 45 and 90 degrees. This thesis shows how the maneuver detection algorithm stopped working when the DF noise exceeded 4.6 degrees for the second scenario and 9.6 degrees for the third scenario.

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Use of an Extended Kalman Filter

by

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN SYSTEMS ENGINEERING
(Electronic Warfare)

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ABSTRACT

The error introduced by the DF accuracy of three Direction Finding equipments used on board two surface units providing bearing measurements is used to evaluate the performance of the ESM systems through the Extended Kalman Filter equations, and to appreciate the result in the estimation process by evaluating how the improvement of the tracking process is reduced as the error introduced by the DF accuracy of the ESM systems is increased.

The process is conducted at three different scenarios involving a change in course of 0, 45 and 90 degrees, and it can be seen how the maneuver detection algorithm stop working when the DF accuracy reaches 4.6 deg. for the second scenario and 9.6 deg. for the scenario Nr 3.

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I. INTRODUCTION

A. DESCRIPTION OF THE SITUATION

One of the drug delivery routes from South and Central America to the United States is conducted by sea. Drugs are very often carried by means of fast boats that operate between these points. These boats are usually equipped with communication equipments and often with radar equipments which are used not in a specific and standard operational mode but with random transmitting.

B. SCENARIO

We will assume that a fast patrol boat transmitting between stations and equipped with radar equipment, is being operated without following a specific and standard transmission plans. Two strategically-situated small patrol boats are going to operate in a passive way such that they do not reveal their position. These boats will use Direction Finding (DF) equipments in the radar bands, which will provide bearings of the received emissions and which will be used to determine the drug boat position from these two bearings. It will track the boat in order to determine the possible future destination of the delivery.

The tracking of the target will be conducted by using the Kalman Filter algorithms.

C. OBJECTIVE OF THIS REPORT

This report will analyze some DF equipments available in the market which might be installed on board patrol boats, in order to determine the accuracy of the trackings using bearings only.

Input considerations will be the available data of DF accuracy or error of the systems as obtained from Reference 1. This will depend on the band of operation of the equipment and the purpose for which it is intended (radar or communications direction finding).

Fast drug boats normally travel at a constant speed and course between the destination points of the deliveries. For the purpose of this analysis, the target is assumed to have a constant speed during the detection and determination of the bearings phase, and will include in some cases a change of course which will affect the accuracy of the tracking process.

II. FORMULATION OF THE PROBLEM

The problem is based on the detection of the transmissions by determining the direction of the emissions.

This situation is shown in Figure 1.

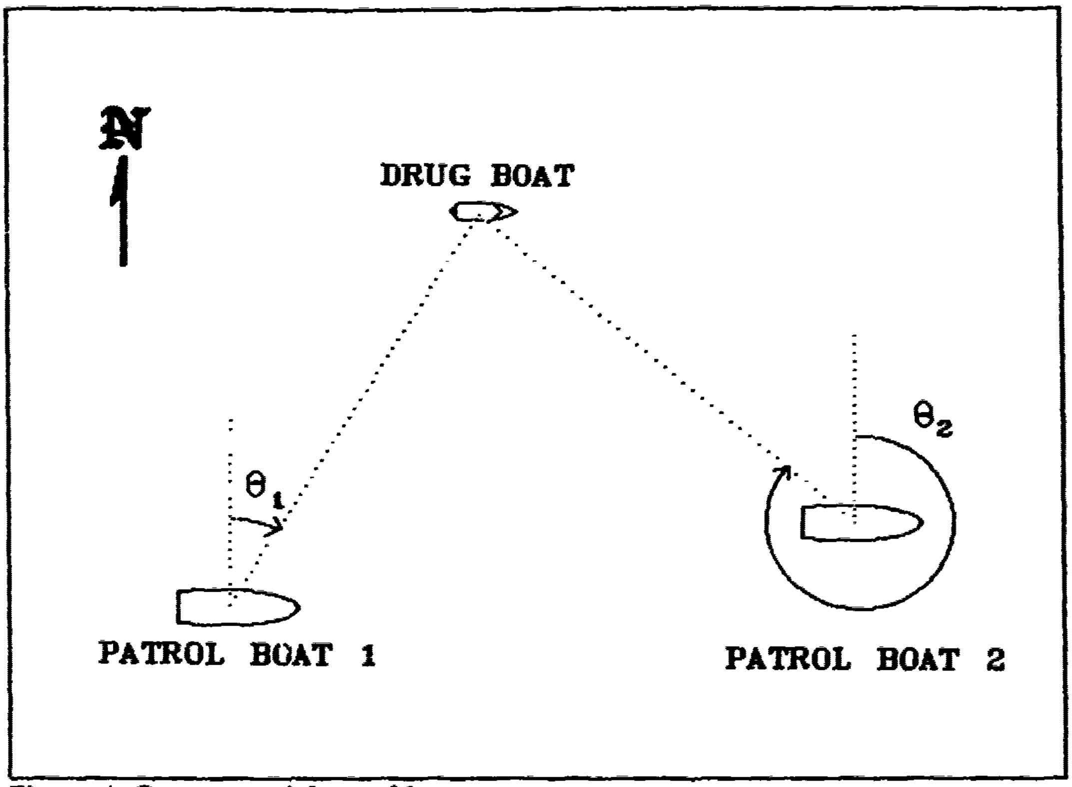

Figure 1. Geometry of the problem

The Kalman filter algorithm will process the received bearings from the two patrol boats and will estimate the target position keeping track of its trajectory.

A. THE SYSTEM MODEL

The two patrol boats will use a reference system for position determination based on X and Y coordinates. The position of the patrol boats is known, as is their course and speed. The desired information about the drug boat is its position and velocities in X and Y coordinates.

The coordinate position is based on the following dynamics:

$$x_k = x_{(k-1)} + \dot{x}T \tag{2-1}$$

$$y_k = y_{(k-1)} + \dot{y}T \tag{2-2}$$

where :

- x : Actual coordinate in X
- y : Actual coordinate in Y
- y_{k-1} : Previous position in X
- y_{k-1} : Previous position in Y
- $\dot{x}$: Velocity component in the X direction
- $\dot{y}$: Velocity component in the Y direction
- T : Time interval of the observation, which represents the time since the previous positions were measured.

In order to estimate these parameters, we need to have a model for the system. In this case, the discrete state-space model representation is given by Equation (2-3) which is a standard state space matrix representation of a linear system of discrete difference

$$x(k+1)=\Phi x(k) \tag{2-3}$$

equations representing the physical state of the system where :

$$x(k)=\begin{bmatrix} x(k) \\ v_x(k) \\ y(k) \\ v_y(k) \end{bmatrix} \tag{2-4}$$

and Φ is the state transition matrix represented by :

$$\Phi=\begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2-5}$$

B. THE MEASUREMENT MODEL

The equations that involve a non-linear measurement process are related to the state variables and can be modeled using the following equation :

$$z=M(x)+v \tag{2-6}$$

where :

z = Measured data (input to the system)

x = State vector

v = Measurement Noise

The measurement data involve the bearings obtained from the DF equipments on board the two patrol boats as stated in Figure 1. This represents a non linear relationship

the measurements and the state variables. The actual measurement equation is as follows :

$$z_k = \tan^{-1}\left[\frac{(x_k - x_{nk})}{(y_k - y_{nk})}\right] + v_k \tag{2-7}$$

where :

z_k = Observed true bearings from patrol boats

x_k, y_k = Position of the drug boat at time k

x_{nk}, y_{nk} = Position of patrol boat "n" at time k

v_k = Measurement noise

This measurement equation should now be linearized in order to be able to work with the linear Kalman filter equations.

C. NOISE CONSIDERATIONS

We will consider the error associated with the bearing accuracy of the DF equipments.

The measurement noise v will be assumed as having zero mean and variance given by the specifications stated in Reference 1. This will constitute the measurement error source.

The bearing accuracy from the specific equipments installed on board the patrol boats were verified.

III. KALMAN FILTER

The application of the principles of modern estimation techniques to multisensor navigation systems began shortly after they were published [Ref. 3]. The Kalman filter presented "a technique for systematically employing all available external measurements, regardless of their errors, to improve the accuracy of navigation systems". [Ref.3: p.5]

The filtering process refers to the estimation of the state vector at the present time based upon present and past measurements. In order to perform its job, the filter needs an a priori knowledge of the state estimate $\hat{x}_{(k/k-1)}$, its error covariance matrix $P_{(k/k-1)}$ and also the actual observation z_k, which in the case of this report, represents the bearings obtained by the DF equipments on board the patrol boats.

Figure 2 shows the entire process to estimate the state of a linear system composed of the "System", "Measurement" and "Kalman Filter" with the sources of errors considered: System error and Measurement error.

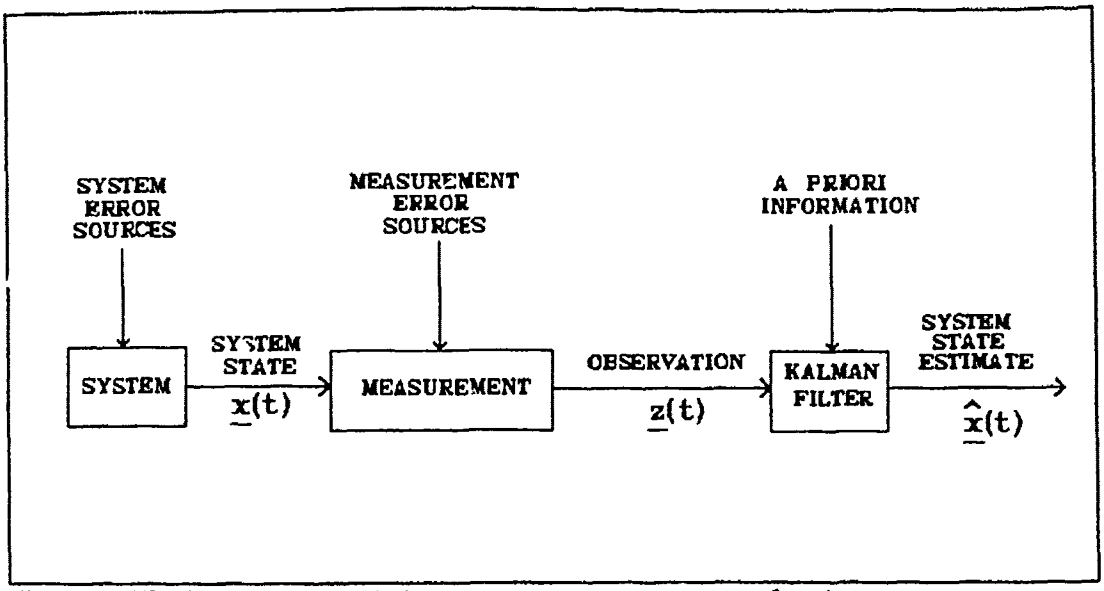

Figure 2. Block diagram depicting system, measurement and estimator

A. EXTENDED KALMAN FILTER

The measur· · equation formulated for the present case (Eq. 2-7) represents, a nonlinear relationsh ·· between the observed bearings and the state variables. In order to use the Kalman filter eq.. tions in this situation, it has to be "adapted" to this nonlinear application. This adaptation of the Kalman filter to a nonlinear application constitutes the Extended Kalman Filter.

1. Linearization

For the linearization process, the Jacobian of the nonlinear measurement equation, has to be formed, so :

$$z_k = h(\underline{x}(k)) + v_k \tag{3-1}$$

where the observation matrix $\mathbf{h}_k$ is a function of the state at each sampling time.

In order to linearize this equation we need to expand **h** in a Taylor series expansion about a estimated trajectory which is continually updated with the filter's estimates.

In this fashion we obtain a first order approximation keeping only the first term in the series expansion.

Taking the Jacobian of Equation 2-7,

$$H_k=\left[\frac{\delta h(x_k)}{\delta x_k}\right] \tag{3-2}$$

and applying the linearization method, we get

$$H_k=\frac{\delta\left[\tan^{-1}\left[\frac{(x_k-x_{nk})}{(y_k-y_{nk})}\right]\right]}{\delta x_k} \tag{3-3}$$

which by simplification gives

$$H_k=[h_{11}\ \ h_{12}\ \ h_{13}\ \ h_{14}] \tag{3-4}$$

where :

$$h_{11}=\frac{\delta\left[\tan^{-1}\left[\frac{(x_k-x_{nk})}{(y_k-y_{nk})}\right]\right]}{\delta x_k}=\frac{(y_k-y_{nk})}{\hat{R}_K^2} \tag{3-5}$$

$$h_{12}=\frac{\delta\left[\tan^{-1}\left[\frac{(x_k-x_{nk})}{(y_k-y_{nk})}\right]\right]}{\delta \dot{x}_k}=0 \tag{3-6}$$

$$h_{13}=\frac{\delta\left[\tan^{-1}\left[\frac{(x_k-x_{nk})}{(y_k-y_{nk})}\right]\right]}{\delta y_k}=-\frac{(x_k-x_{nk})}{\hat{R}_k^2} \tag{3-7}$$

$$h_{14}=\frac{\delta\left[\tan^{-1}\left[\frac{(x_k-x_{nk})}{(y_k-y_{nk})}\right]\right]}{\delta\dot{y}_k}=0 \tag{3-8}$$

By doing some substitutions in equations (3-5) and (3-7):

$$x_k=\hat{x}_{k/k-1}$$

$$y_k=\hat{y}_{k/k-1}$$

the linearized measurement matrix can be written as :

$$H_k=\left[\frac{(\hat{y}_{k/k-1}-y_{nk})}{\hat{R}_k^2} \quad 0 \quad \frac{-(\hat{x}_{k/k-1}-x_{nk})}{\hat{R}_k^2} \quad 0\right] \tag{3-9}$$

where the range from each sensor (patrol boat) to the target (drug smugglers boat) is computed by :

$$\hat{R}^2=(\hat{y}_{k/k-1}-y_{nk})^2+(\hat{x}_{k/k-1}-x_{nk})^2 \tag{3-10}$$

Once the measurement equation is linearized about $\hat{x}_{(k/k-1)}$ which represents the updated state estimate at time k, based on the previous estimates calculated at time K-1, the normal linear Kalman filter equations can be applied to solve the problem.

2. Noise Processes

In order to compute the error covariance matrix, the filter needs to know the covariance matrix of the measurement noise process v_k.

$$E\left[v_k v_k^T\right]=R_k \tag{3-11}$$

where R_k is defined as the state measurement noise covariance matrix, and it is based on the accuracy of the DF equipments.

The excitation covariance matrix Q_k, involves the stochastic model of accelerations of the target.

The geometry of the target will be represented as specified in the Figure 3,

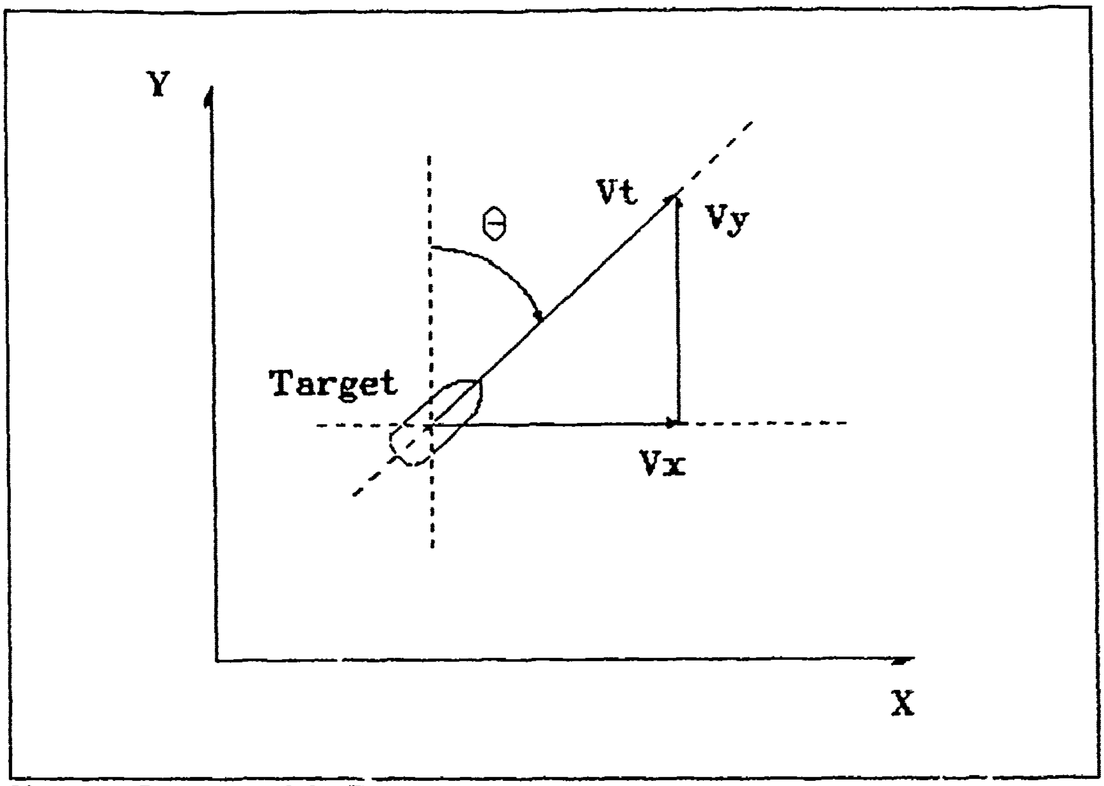

Figure 3. Geometry of the Target

where it can be seen that the velocity of the target can be described as:

$$V_x = V\sin(\theta) \tag{3-12}$$

$$V_y = V\cos(\theta) \tag{3-13}$$

By taking the time derivative of these two equations we get the target's acceleration in its x and y components

$$a_x=\dot{V}\sin(\theta)+V\dot{\theta}\cos(\theta)$$

$$=\dot{V}\frac{V_x}{V}+V\dot{\theta}\frac{V_y}{V} \qquad (3\text{-}14)$$

$$=\dot{V}\frac{V_x}{V}+\dot{\theta}V_y$$

$$a_y=\dot{V}\cos(\theta)-V\dot{\theta}\sin(\theta)$$

$$=\dot{V}\frac{V_y}{V}-V\dot{\theta}\frac{V_x}{V} \qquad (3\text{-}15)$$

$$=\dot{V}\frac{V_y}{V}-\dot{\theta}V_x$$

Assuming a linear and angular accelerations having a zero mean

$$E[\dot{V}]=E[\dot{\theta}]=0 \qquad (3\text{-}16)$$

the variances becomes :

$$E[\dot{V}^2]=\sigma_V^2 \qquad (3\text{-}17)$$

$$E[\dot{\theta}^2]=\sigma_\theta^2 \qquad (3\text{-}18)$$

The assumed values for the linear and angular accelerations were taken as:

$\sigma_V^2=0.0005$ [illegible] (3-19)

$$\sigma_\theta^2=0.01\left(\frac{rad}{hr^2}\right)^2 \qquad (3\text{-}20)$$

In order to calculate the state excitation covariance matrix Q_k, we need to have the variances of the accelerations in the x and y directions, so taking the expected value of the Equations (3-14) and (3-15) we will have :

$$E[a_x^2] = \text{[illegible]} \sigma_V^2 + V_y^2 \sigma_\theta^2 \tag{3-21}$$

$$E[a_y^2] = \text{[illegible]} \sigma_V^2 + V_x^2 \sigma_\theta^2 \tag{3-22}$$

and the covariance of a_x and a_y is equal to :

$$E[a_x a_y] = E[a_y a_x] = V_x V_y \left[\left(\frac{\sigma_V}{V} \right)^2 - \sigma_\theta^2 \right] \tag{3-23}$$

With all these calculations the value of the state excitation covariance matrix becomes :

$$Q_k = \left[\Gamma_k Q_k' \Gamma_k^T \right] \tag{3-24}$$

where Γ_k is the system noise coefficient matrix and is represented by :

$$\Gamma_k = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix} \tag{3-25}$$

and Q' is equal to :

$$Q' = \begin{bmatrix} E[a_x^2] & E[a_{xy}] \\ E[a_{yx}] & E[a_y^2] \end{bmatrix} \tag{3-26}$$

3. Initialization of The Extended Kalman Filter

For the purpose of obtaining the best and accurate estimates from the Kalman filter process, the filter must be initialized with an Initial State Estimate and an Initial Error Covariance matrix. If this initial state estimate is far from the real target position, the linearization effects from the nonlinear measurement equation will cause the filter to diverge, giving an erroneous output.

The initialization of the filter will be based on the intersection of the first two received bearings. The estimated initial target position will be calculated as :

$$x=\left[-\frac{y_{s2}\tan(\theta_2)+y_{s1}\tan(\theta_1)+x_{s2}-x_{s1}}{\tan(\theta_1)-\tan(\theta_2)}-y_{s1}\right]\tan(\theta_1)+x_{s1} \tag{3-27}$$

$$y=-\frac{y_{s2}\tan(\theta_2)+y_{s1}\tan(\theta_1)+x_{s2}-x_{s1}}{\tan(\theta_1)-\tan(\theta_2)} \tag{3-28}$$

Since no information is available on the course and speed of the target at the initialization moment that could help in the estimation of the velocities of the target in X and Y directions, they are taken as zero. Figure 4 represents the initialization of the initial position estimate.

This initial position estimate and the assumption of zero velocity in the x and y directions include some error that can be assumed to be within some standard deviation. The estimate of the errors of this initial estimates will be used to construct the Initial Error Covariance matrix.

Following the approach of Bennett [Ref. 4] and Galinis [Ref.5], the value of an initial standard deviation of position error is taken as 100 nautical miles both in the x and y directions, and the initial velocity standard deviation is taken to be 0.5 nautical miles per

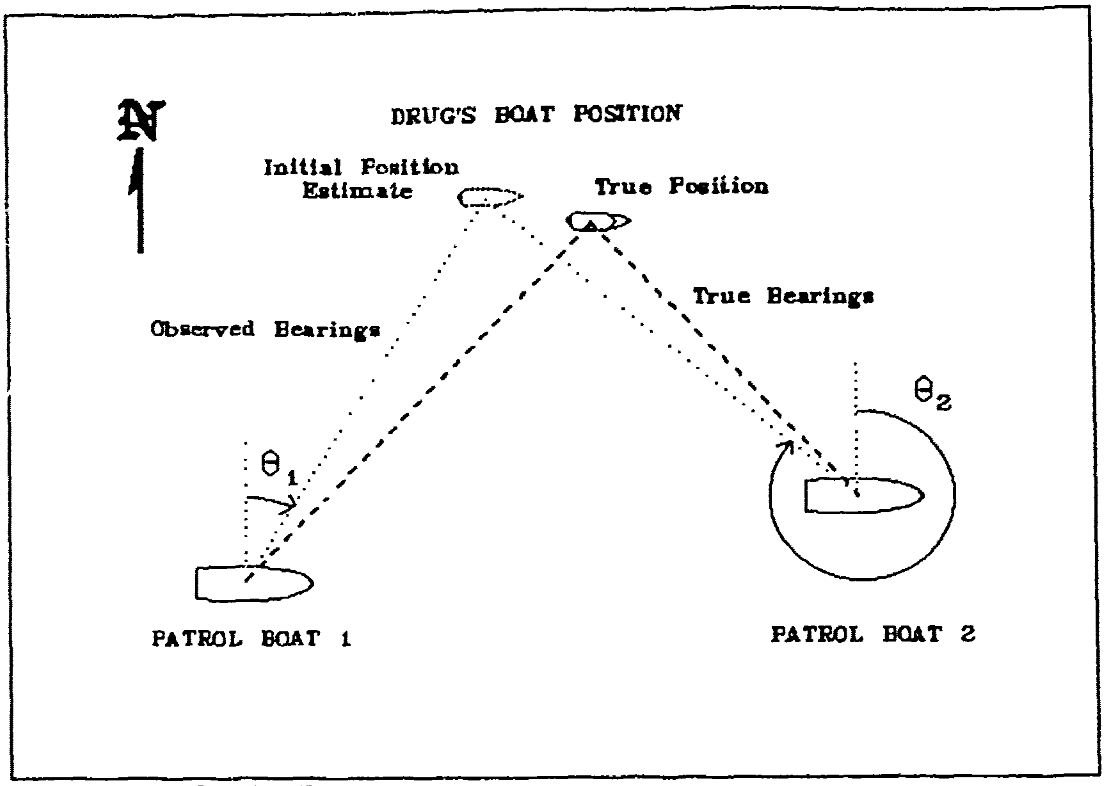

Figure 4. Initialization Process

minute which is equal to a 30 knots speed. Both errors are assumed to be uncorrelated and have zero mean.

The resultant Initial Error Covariance matrix for time k=0, can be written as :

$$P(k/k-1)=P(0/-1)=\begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix} \qquad (3\text{-}29)$$

Now that the filter is initialized the estimation process of the target position is ready to begin.

4. Extended Kalman Filter Operation

For the continuous operation of the filter, the observation bearings are simultaneously and continuously received from the two sensors at a time interval T equal to the time difference in minutes between the observations.

The A Priori State Estimate $X_{(k/k-1)}$ and the State Error Covariance Matrix $P_{(k/k-1)}$ are computed by using the following Equations :

$$\hat{x}_{k+1/k}=\hat{x}_{k/k-1}=\Phi\hat{x}_{k/k} \tag{3-30}$$

$$P_{k/k-1}=\Phi P_{k/k}\Phi^{T}+Q \tag{3-31}$$

Once the A priori or Projected State Estimate is calculated, it is used to compute the Linearized Observation Matrix H_k in Eq.(3-9), and with all these together the Kalman Gain matrix G_k is computed as follows:

$$G_k=P_{k/k-1}H_k^T(H_kP_{k/k-1}H_k^T+R_k)^{-1} \tag{3-32}$$

The Kalman Gain matrix represents the confidence given by the filter to a priori information with respect to the current observation. It minimizes the square estimation error and indicates how much weight will be placed on the current observation. If $P_{(k/k-1)}$ is relatively small, the Kalman Gain matrix will be close to zero due to the finite value of R_k. If $P_{(k/k-1)}$ is relatively large the Kalman Gain matrix will be close to one, so the Kalman Gain is proportional to the uncertainty in the estimate $P_{(k/k-1)}$ and inversely proportional to the measurement noise R_k.

The Kalman Gain matrix will directly affect the calculation of the state estimate as it is seen in Eq.(3-17)

$$\hat{x}_{k/k}=(I-G_kH_k)\hat{X}_{k/k-1}+G_kz_k \quad (3\text{-}33)$$

where the Kalman Gain matrix affects the weight placed on the current observation z_k. "A large Gain, indicating a large error covariance, will place more weight on the current observation as the filter tries to correct the states. A small gain, indicating a small error covariance, places less emphasis in the new observation" [Ref.5: p.15].

The Error covariance matrix will also be updated by considering the Kalman Gain matrix as it is stated in the next equation:

$$P_{k/k}=(I-G_kH_k)P_{k/k-1} \quad (3\text{-}34)$$

The process then will repeat itself by computing the next observation from the two sensors, and the continuous estimation process of this information will allow the filter to keep track of the target movements and maneuvers.

B. MANEUVER DETECTION

For the present work, the target is assumed to have a constant course and speed for the initial scenario and then it is included to have a change of course in order to evaluate the response and the ability of the filter to keep the tracking process for the evaluation of each independent DF system.

In order to be able to keep track of the target once a maneuver has occurred, the filter has to use some kind of detection system.

The use of the residual bias as a maneuver detector, as proposed by McAulay and Denlinger [Ref. 6] and followed by Bennett [Ref.4], will be used in the present thesis by applying a second-order moving average filter as the residual bias.

The output of the bias filter is:

$$M_k=\frac{1}{3}\left[e_k+e_{k-1}+e_{k-2}\right] \tag{3-35}$$

where the observation residual e_k is defined as :

$$e_k=z_k-H(\hat{x}_{k/k-1}) \tag{3-36}$$

The window provided by the moving average filter is wide enough to absorb a large amount of error in the bearing observations that are far outside the standard deviation, but narrow enough to detect a maneuver as soon as it has actually occurred.

When the mean of the residual process exceeds a maneuver detection threshold, the filter determines that a maneuver has occurred and resets the filter parameters in order to maintain an accurate track until the following maneuver is detected. This detection threshold or GATE is chosen to be 1.5 times the standard deviation of the residual process

$$GATE=1.5\sqrt{H_kP_{K/K-1}H_k^T+R_k} \tag{3-37}$$

which as stated in Reference 6 will result in a 90% probability of detection of the maneuver with a 10% probability of false alarm rate.

IV. DRUG SMUGGLERS AT SEA

The drug traffic at sea is conducted using boats and electronic equipment available in the commercial area. The military services have been tasked to counter these organizations.

The US Department of Defense established two control centers for drug interdiction in the United States. In February 1989 the Joint Task Force 4 was established in Key West, Florida and Joint Task Force 5 in Alameda, California, each of them having the responsibility of anti-drug operations in their respective operational areas, Caribbean and Pacific.

The boats and ships used for the drug delivery purposes are commercial and recreational and are equipped with navigation radars available anywhere in the market. Those belonging to the FURUNO company are the most popular. This radar will be used on the present thesis with the characteristics and information available from the FURUNO product catalog [Ref. 7].

A. RADAR CHARACTERISTICS

The radars installed could be one of several available from this company (1700,1800,1900, FR-7000D,FR-8000D,FR-1500D,FR-2000 or CR-900) which operate in the X and S bands at different output powers for different ranges.

We will assume that the equipment belongs to the FR-8000D Series radars from FURUNO (model FR-8100DS), which has the following characteristics, as it is stablished in Reference 7 :

- Navigation radar
- S-band operation (2-4 Ghz.)
- High resolution : 640 x 480 pixel 12" CRT display
- High accuracy : 0.9 % of range in use and 0.2 degree bearing resolution
- 10 Kw of output power
- No compromise 8-level quantization, coupled with a MIC low noise receiver, four transmitter pulselengths, three pulse repetition rates and two receiver IF bandwidths
- Range : 1/4 to 72 nautical miles
- Improved detection by Echo Stretch, Echo Averaging, Interference Rejector, Sea & Rain Clutter Control

V. DIRECTION FINDING EQUIPMENTS TO BE ANALYZED

The selected DF equipments will be considered to evaluate the performance of the estimation process for each equipment. The characteristics of them will be specified as follows as taken from Jane's [Ref. 1:pp. 346-352].

A. GUARDIAN STAR SHIPBORNE EW SYSTEM

- Shipborne radar detection and surveillance system
- Frequency range : 2-18 GHz
- The system consists basically of an antenna assembly with an onmi-directional and six spiral DF antennas, an RF/digital interface unit and a display/controller unit
- Versions : The MK 1 system offers early warning to small surface craft or patrol boats with a bearing accuracy provided by octave frequency measurements. The MK 2 system is designed to meet the basic ESM needs of surface ships and submarines providing YIG tuned frequency. The MK 3 is an ELINT system for surface ships and submarines, providing instant threat warning and accurate frequency measurement by IFM devices in the receiver front end
- Can store up to 2000 emitter parameters in the library
- DF accuracy : 2.5 degrees in the 2-8 GHz band
1.5 degress in the 8-18 GHz band
- Operational status : In production
- Contractor : Sperry Corporation, Arlington, Virginia, USA

B. ELETTRONICA SpA. EW EQUIPMENT

- Shipborne integrated EW system
- Frequency range : 2-18 GHz

- Based on a four-band instantaneous frequency measuring (IFM) receiver which perform all ESM functions

- Components : ELT/116 Radar intercept receiver, ELT/711 Radar identification unit, ELT/712 Programming unit, ELT/716 Data transmission module, ELT/311/511 Jammer

- The ELT/116 radar intercept receiver system includes omnidirectional and DF antennas, an auxiliary DF unit, RF unit and display.The IFM receiver operates with a crystal video DF receiver in all bands

- Frequency measurement accuracy : 0.2 % with a Dynamic range of about -70 dBm to zero

- DF accuracy : 7 degrees

- Operational status : In production and in operational use

- Contractor : Elettronica SpA, Rome, Italy

C. RDL-1BC ESM EQUIPMENT

- Shipborne radar surveillance and detection system

- Frequency range : 2-11.5 GHz

- The system is a basic tactical small ship ESM system, suitable to use on board fast patrol boats

- Provides instantaneous bearing, automatic pulse analysis (APA-1C Pulse Analyzer) and alarm together with measurement of frequency band

- DF accuracy : 20 degrees

- Operational status : In service but no longer in production

- Contractor : Racal Radar Defence Systems Ltd. Chessington, England

VI. SIMULATIONS

The programs here are based on those developed by Spehn [Ref. 2], Bennett [Ref. 4] and Galinis [Ref. 5] which are written in Microsoft FORTRAN, and MATLAB. The following especifications should be followed when desired to run.

- TRKDATA.FOR : Will generate the data needed for the calculation and estimation process, having the option of using noisy or no-noise bearing observations from two observers. In order to run the program in the noisy option, a file must be generated containing the noise from a random number generator which should consider the variance of the error associated in this case with the DF accuracy of the ESM systems. The output of this program is stored in the file TRKDATA containing the TIME, X and Y coordinates of the sensors and the Bearings from each sensor. This file is needed to run the main program SHIPTRACK.FOR.

- SHIPTRACK.FOR : Will read the file TRKDATA and based on the Extended Kalman filter equations, will calculate the estimated position of target, producing the following output files : OUTDATA which contains the time, estimated X and Y coordinates of the target, calculated X and Y coordinates of the target based on the bearings measurements from sensors 1 and 2. TRKERR containing the time, tracking and observed errors, and TRKINFO which contains the time, and the estimated parameters of the target : X and Y coordinates, course and speed.

- PLOT.M : A MATLAB function file created to graph the results obtained from the simulations. For this purpose, the output files should be transferred to a MATLAB subdirectory that already contains this function file.

A. DESCRIPTION OF THE SCENARIOS

In order to see the effects of the tracking process by the Kalman filter algorithm, three different scenarios will be considered. The drug smugglers boat is transmitting between its delivery stations and is being monitored by two patrol boats which obtain bearings from the transmission of the radar installed on board the drug boat.

For all simulations, the patrol boat travels at the same course and at a speed of 16 knots (a very reasonable cruising speed for a ship), so that no suspicion is produced.

Each situation will be analyzed with the three DF equipments previously described in Chapter V. The performance of each of them will be evaluated.

Three scenarios are shown with the following characteristics:

1. **Scenario Nr.1**

- Drug boat traveling at course 090 degrees at 30 knots
- Patrol boat 1 traveling at course 060 at 16 knots
- Patrol boat 2 traveling at course 350 at 16 knots
- This scenario is shown in Figure 5

2. **Scenario Nr. 2**

- Drug boat traveling at course 090 degrees at 30 knots until time t=15 minutes when it maneuvers assuming course 045 maintaining a speed of 30 knots
- Patrol boat 1 traveling at course 060 at 16 knots
- Patrol boat 2 traveling at course 350 at 16 knots
- This scenario is shown in Figure 6

3. **Scenario Nr. 3**

- Drug boat traveling at course 090 degrees at 30 knots until time t=15 minutes when it maneuvers assuming course 000 maintaining a speed of 30 knots
- Patrol boat 1 traveling at course 060 at 16 knots
- Patrol boat 2 traveling at course 350 at 16 knots
- This scenario is shown in Figure 7

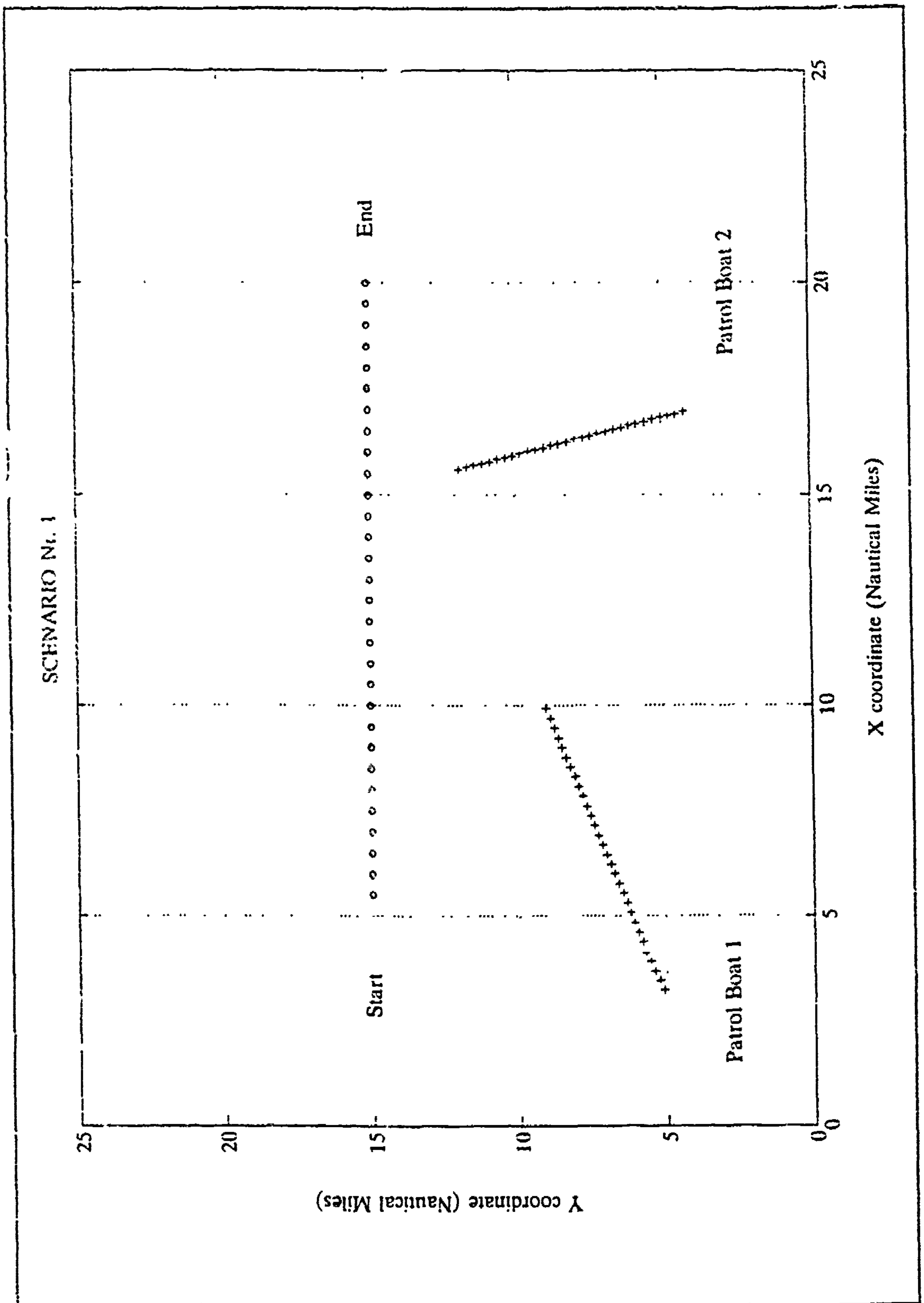

Figure 5. Scenario Nr. 1

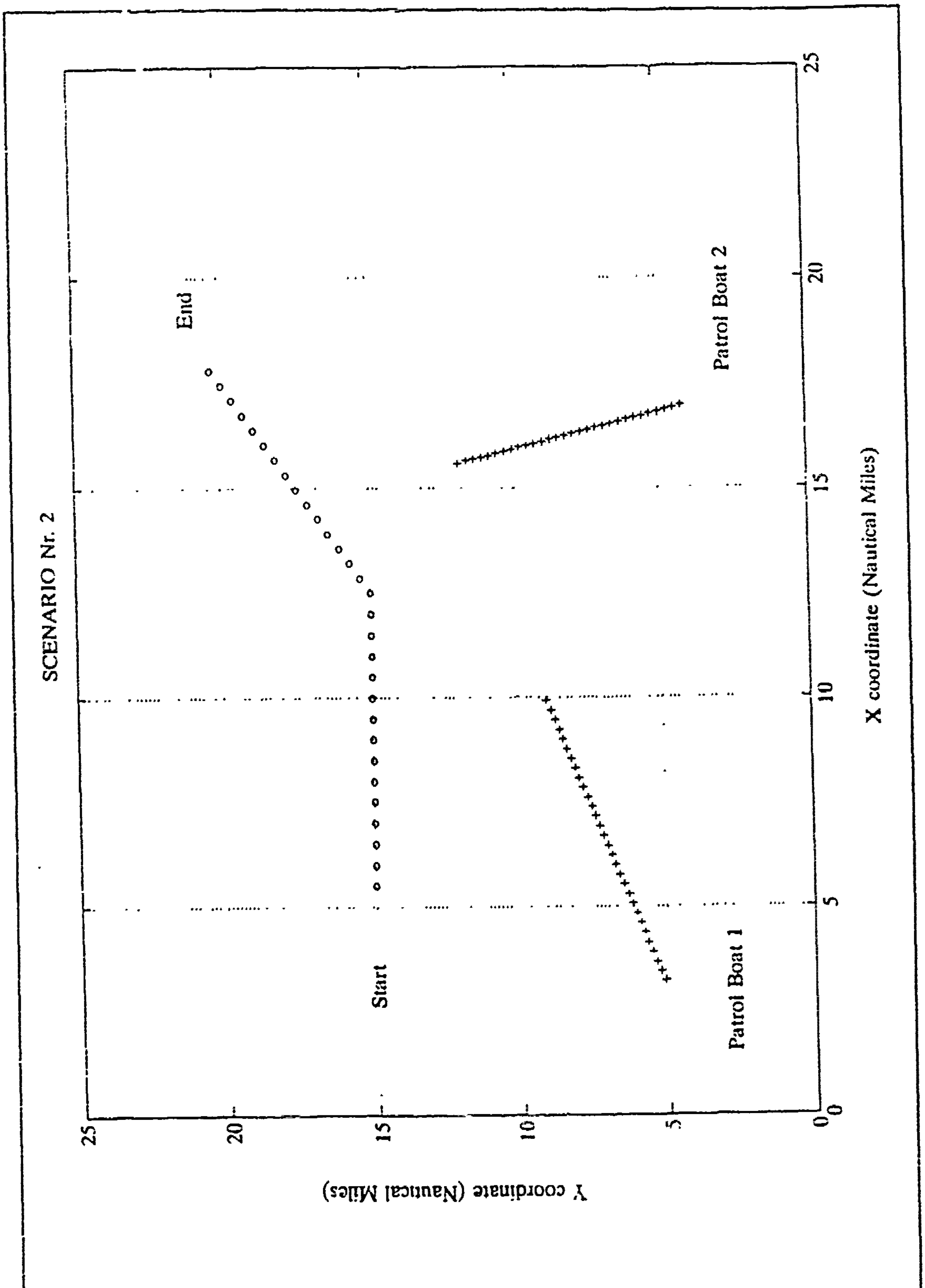

Figure 6. Scenario Nr. 2

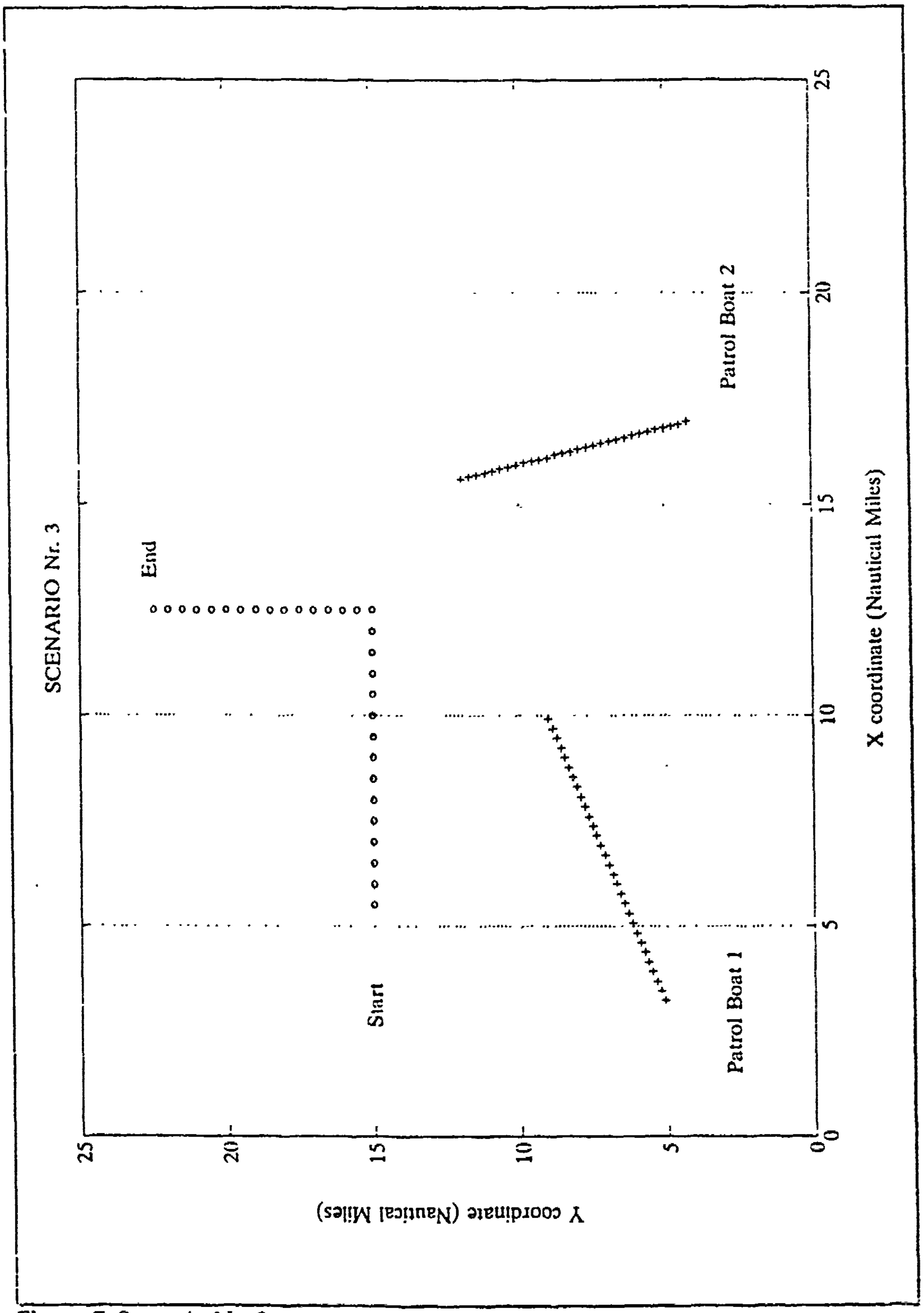

Figure 7. Scenario Nr. 3

B. RESULTS OF THE SIMULATIONS

1. GUARDIAN STAR SHIPBORNE EW SYSTEM

a. *Scenario Nr 1.*

With a DF accuracy of 2.5 degrees, the filter will keep following the target movements very close to its real positions. The maximum tracking error is 0.32 nm maintaining an average of 0.14 nm The results of this scenario are shown in Figures 8 and 9, and the output file TRKINFO containing the target's estimated data is shown below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.378387	14.905070	0.00000	0.00000
1	5.763358	14.938050	88.170590	13.833230
2	6.526980	15.073220	83.468400	31.097350
3	7.145387	15.114900	84.569790	33.608130
4	7.309046	14.965760	91.372160	25.715370
5	7.934094	15.007730	89.623790	28.892010
6	8.546397	15.039240	88.955980	30.698860
7	8.781705	14.955550	91.143860	27.387130
8	9.302892	14.962130	90.782200	28.072280
9	9.900760	14.988120	90.143300	29.308670
10	10.361610	14.984620	90.179310	29.069630
11	10.865720	14.990180	90.058300	29.224970
12	11.338580	14.996020	89.951840	29.120680
13	11.678660	15.012880	89.684380	28.131570
14	12.339770	15.018430	89.622230	29.356790
15	12.848020	15.027450	89.532430	29.471050
16	13.327280	15.045920	89.329100	29.405160
17	13.819690	15.063880	89.154300	29.419710
18	14.411500	15.026960	89.685640	29.935990
19	15.003690	14.962570	90.491620	30.390950
20	15.475100	15.005990	89.897880	30.227580
21	15.960590	15.042060	89.452220	30.147910
22	16.417440	15.162740	87.958690	29.971100
23	16.945640	15.091350	89.122680	30.075770
24	17.465870	14.986290	90.604700	30.147870
25	17.973440	14.835460	92.532780	30.190030
26	18.481840	14.931920	90.927700	30.189910
27	19.010840	15.152880	87.744030	30.301360
28	19.519140	15.189950	87.507160	30.318730
29	20.011280	15.137270	88.628120	30.259360

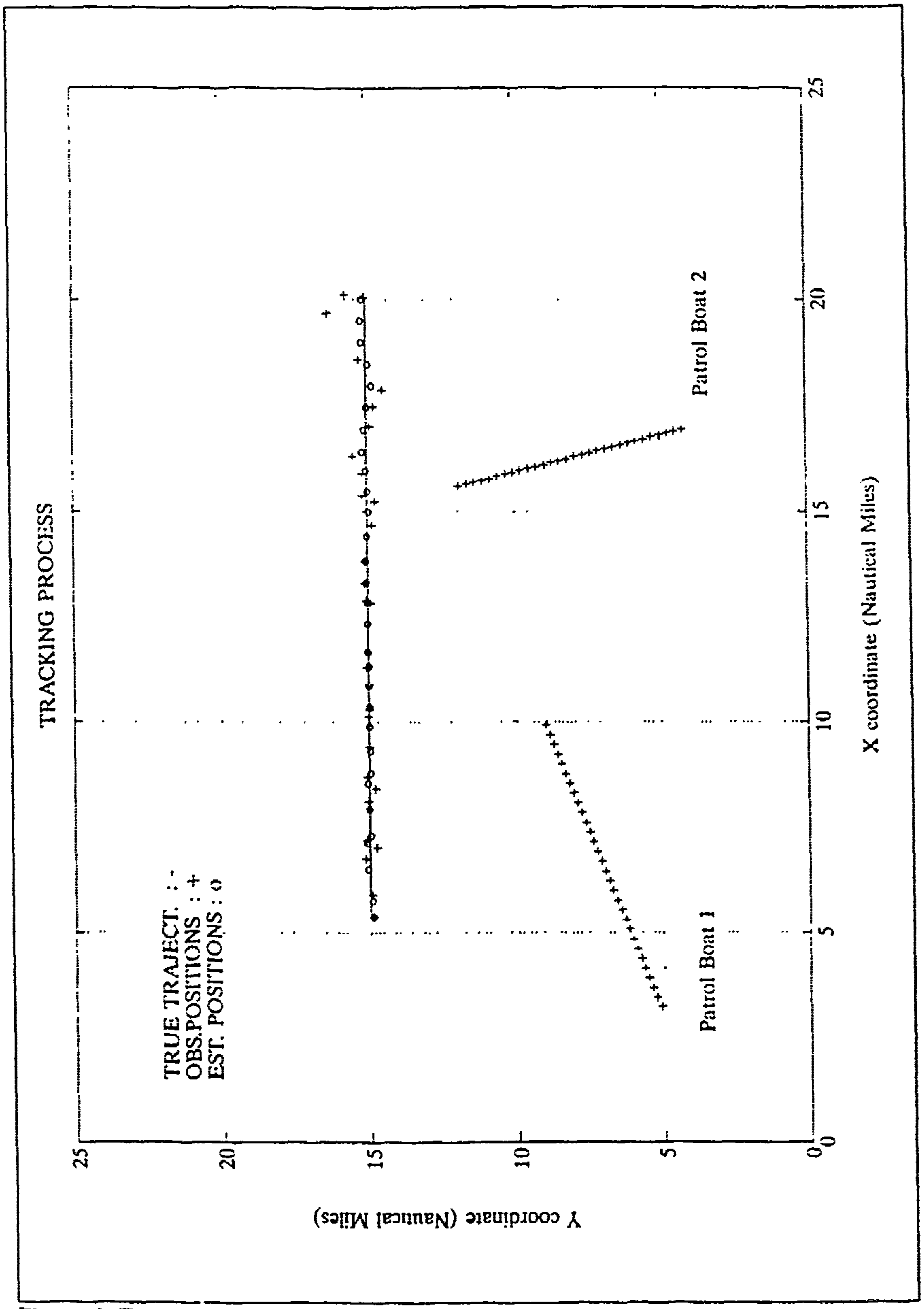

Figure 8. Target tracking

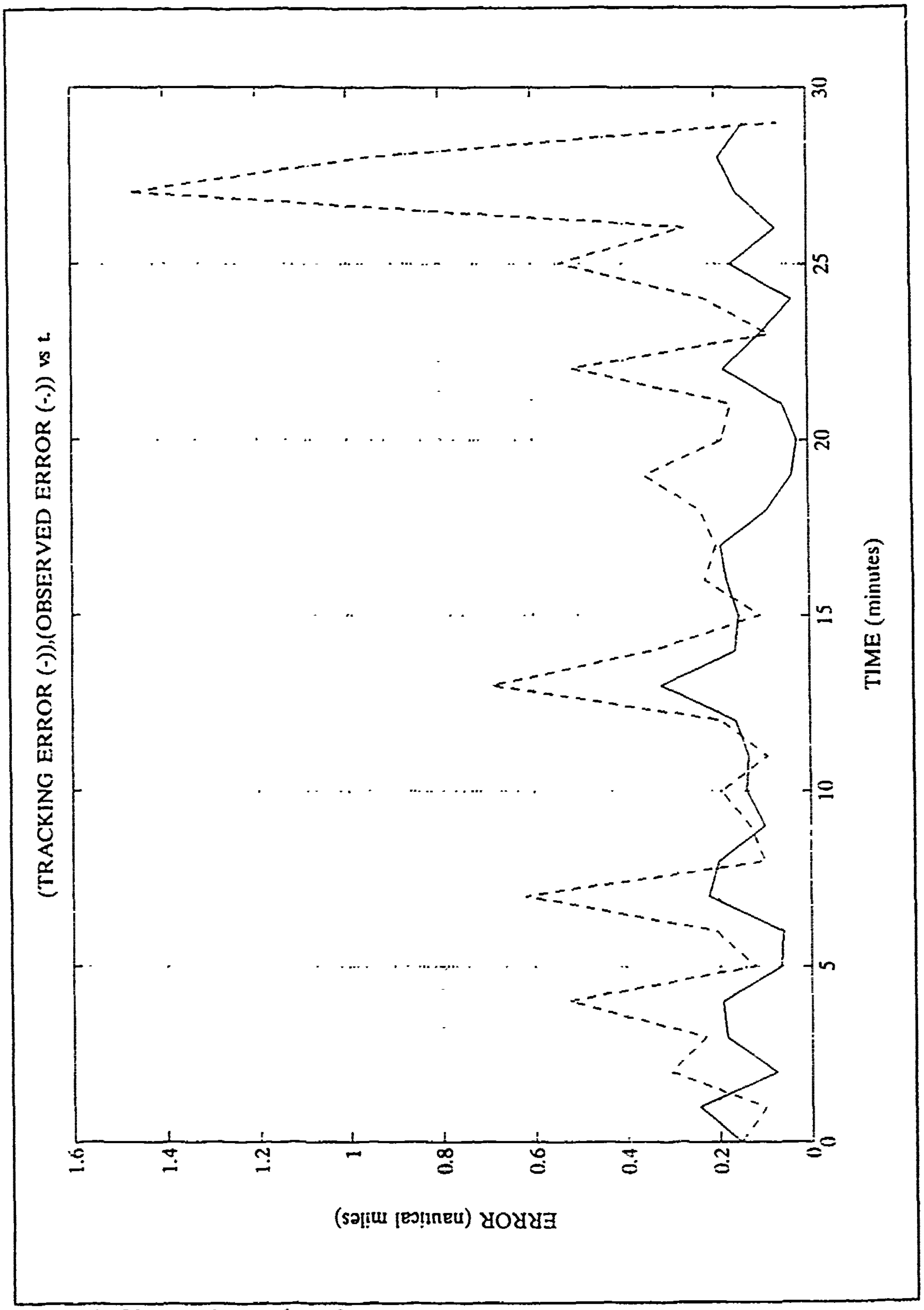

Figure 9. Observation and tracking errors

b. Scenario Nr 2.

For this scenario, the filter will follow the target maneuver to course 045 degrees by evaluating the next 7 observations after the maneuver is produced. This induces a tracking error that reaches 0.94 nm after the maneuver takes place, which is reduced to about 0.25 nm when the maneuver gating of the filter corrects the estimation process. The results of this scenario are shown in Figures 10 and 11, and the output file TRKINFO containing the target's estimated data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.378387	14.905070	0.000000	0.000000
1	5.763358	14.938050	88.170590	13.833230
2	6.526980	15.073220	83.468400	31.097350
3	7.145387	15.114900	84.569790	33.608130
4	7.309046	14.965760	91.372160	25.715370
5	7.934094	15.007730	89.623790	28.892010
6	8.546397	15.039240	88.955980	30.698860
7	8.781705	14.955550	91.143860	27.387130
8	9.302892	14.962130	90.782200	28.072280
9	9.900760	14.988120	90.143300	29.308670
10	10.361610	14.984620	90.179310	29.069630
11	10.865720	14.990180	90.058300	29.224970
12	11.338580	14.996020	89.951840	29.120680
13	11.678660	15.012880	89.684380	28.131570
14	12.339770	15.018430	89.622230	29.356790
15	12.810740	15.116240	88.227130	29.262770
16	13.223690	15.284670	85.992000	28.902960
17	13.625840	15.496370	83.402250	28.610620
18	14.106980	15.693230	81.397750	28.798490
19	14.577360	15.902150	79.511890	28.962680
20	14.921810	16.231590	76.159880	28.764900
21	14.836720	17.593810	47.291450	31.042000
22	15.041560	18.031290	31.958180	27.265090
23	15.608130	18.298390	50.397030	31.575780
24	16.114400	18.513630	56.217260	31.943490
25	16.603260	18.675180	60.262940	31.517530
26	16.884370	19.226940	51.627830	31.718100
27	17.090610	19.950330	42.344990	33.174030
28	17.420320	20.380020	41.441830	33.107540
29	17.805230	20.662320	43.124460	32.322650

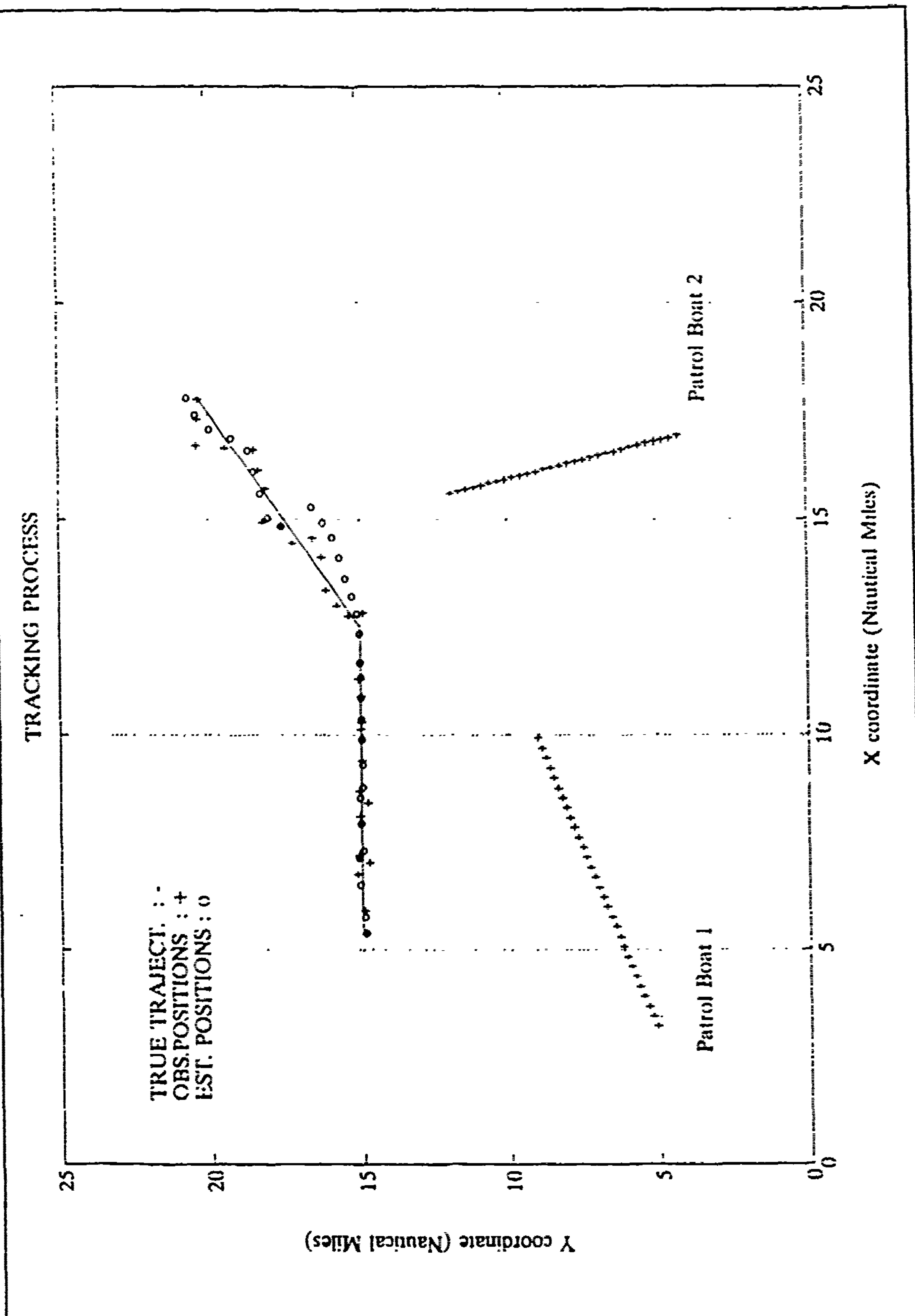

Figure 10. Target tracking

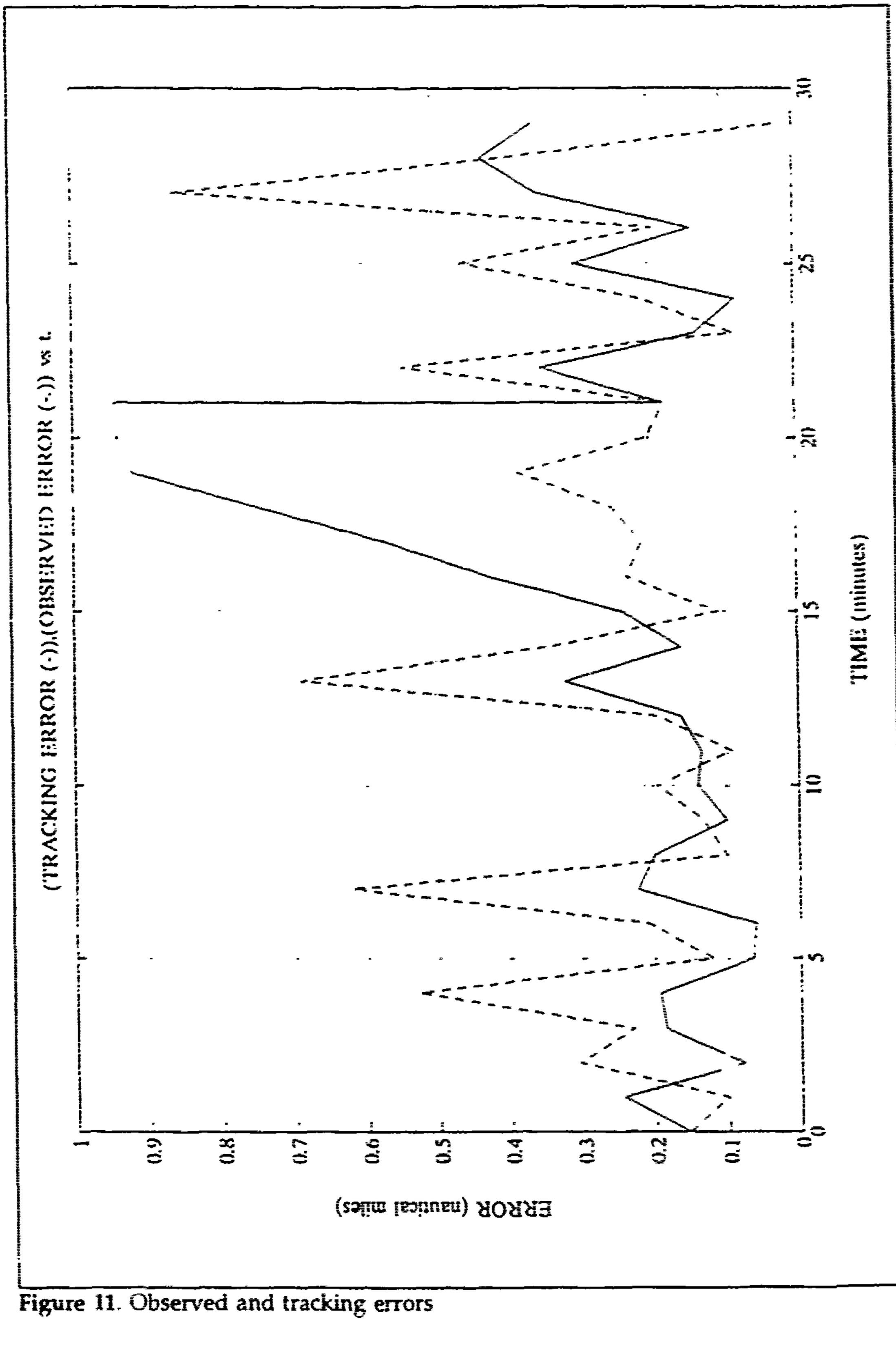

Figure 11. Observed and tracking errors

c. *Scenario Nr 3.*

With a DF accuracy of 2.5 degrees, the filter will evaluate three more observations before detecting the target maneuver. The tracking error reaches 1.05 nm while the maneuver is not detected it is reduced to about 0.1 nm by the maneuver gating after it detects the change of course, keeping a close track of the target movements. The results for this scenario are shown in Figures 12 and 13, and the output file TRKINFO containing the target's estimated data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.378387	14.905070	0.000000	0.000000
1	5.763358	14.938050	88.170590	13.833230
2	6.526980	15.073220	83.468400	31.097350
3	7.145387	15.114900	84.569790	33.608130
4	7.309046	14.965760	91.372160	25.715370
5	7.934094	15.007730	89.623790	28.892010
6	8.546397	15.039240	88.955980	30.698860
7	8.781705	14.955550	91.143860	27.387130
8	9.302892	14.962130	90.782200	28.072280
9	9.900760	14.988120	90.143300	29.308670
10	10.361610	14.984620	90.179310	29.069630
11	10.865720	14.990180	90.058300	29.224970
12	11.338380	14.996020	89.951840	29.120680
13	11.678660	15.012680	89.684380	28.131570
14	12.339770	15.018430	89.622230	29.356790
15	12.718770	15.[illegible]48420	87.738750	28.720650
16	12.976850	15.380340	84.482090	27.586280
17	12.290310	16.546470	2.415755	29.750290
18	12.562290	17.020110	22.479660	31.960900
19	12.832930	17.464880	26.589940	31.743030
20	12.607800	18.005310	7.791852	29.901250
21	12.490300	18.509500	1.641888	29.800140
22	12.266730	19.033420	355.196900	30.262980
23	12.369590	19.539770	358.533500	30.214490
24	12.478760	20.025980	0.982643	30.049860
25	12.623070	20.494540	3.427758	29.837520
26	12.558350	21.007620	1.898402	29.[illegible]4280
27	12.369090	21.540510	358.897300	30.059630
28	12.319710	22.043870	358.367900	30.058670
29	12.347500	22.550470	358.887900	30.127010

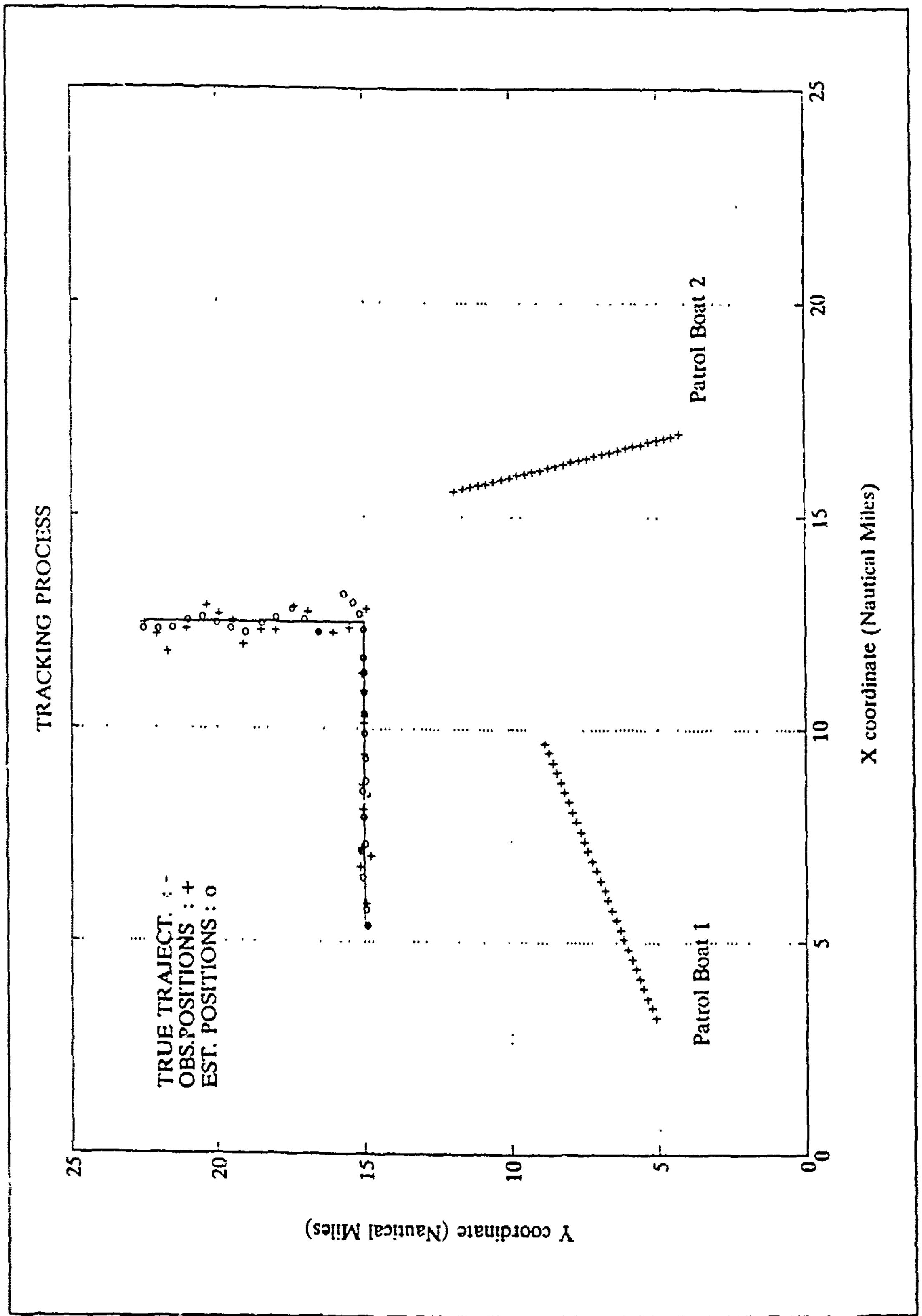

Figure 12. Target tracking

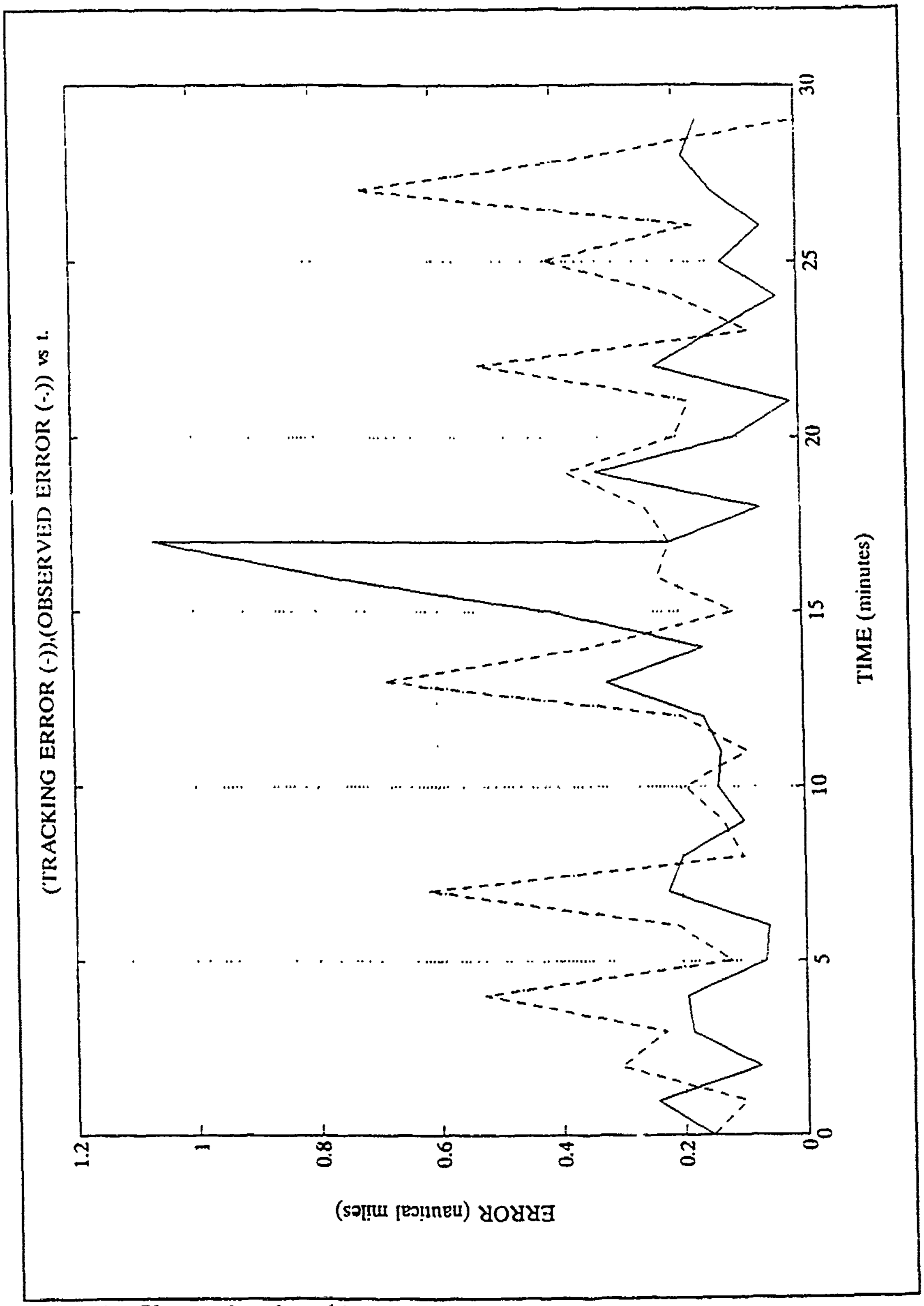

Figure 13. Observed and tracking errors

2. ELETTRONICA SpA. EW EQUIPMENT

a. *Scenario Nr 1.*

With a DF accuracy of 7 degrees, the observations obtained from this EW equipment allow the filter to follow the target's real trajectory in a very close range having an average tracking error of 0.25 nautical miles throughout the process. The results for this scenario are shown in Figures 14 and 15, and the output file TRKINFO containing the target's estimated data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.297562	14.839560	0.000000	0.000000
1	5.606289	14.889010	87.872260	3.000554
2	6.251655	15.076120	84.202790	13.723970
3	6.878047	15.129430	85.245830	21.320830
4	6.996462	14.959290	92.128600	17.180810
5	7.691809	15.012230	89.670380	23.334330
6	8.393325	15.057860	88.702640	27.468300
7	8.522616	14.936160	91.944220	23.643440
8	9.073889	14.943670	91.379620	25.277470
9	9.745502	14.987170	90.270650	27.624500
10	10.196570	14.979060	90.378960	27.544940
11	10.714290	14.987440	90.189420	28.005710
12	11.179040	14.995240	90.040020	27.990760
13	11.423670	15.014810	89.716520	26.488830
14	12.195530	15.038620	89.425810	28.582370
15	12.714280	15.051680	89.317120	28.835680
16	13.184210	15.078110	89.059140	28.778160
17	13.675140	15.104270	88.839810	28.841620
18	14.332260	15.052760	89.486910	29.734760
19	14.990810	14.957260	90.464160	30.525880
20	15.445360	15.017140	89.851720	30.273840
21	15.922470	15.064400	89.418130	30.153260
22	16.347350	15.223530	87.984070	29.836090
23	16.897680	15.133180	88.968120	30.046650
24	17.439610	14.998850	90.238110	30.210470
25	17.966550	14.798780	91.953250	30.322900
26	18.470410	14.911470	90.837410	30.299770
27	18.995870	15.192560	88.360760	30.368650
28	19.508590	15.249040	88.002770	30.395650
29	20.001910	15.191520	88.658020	30.348060

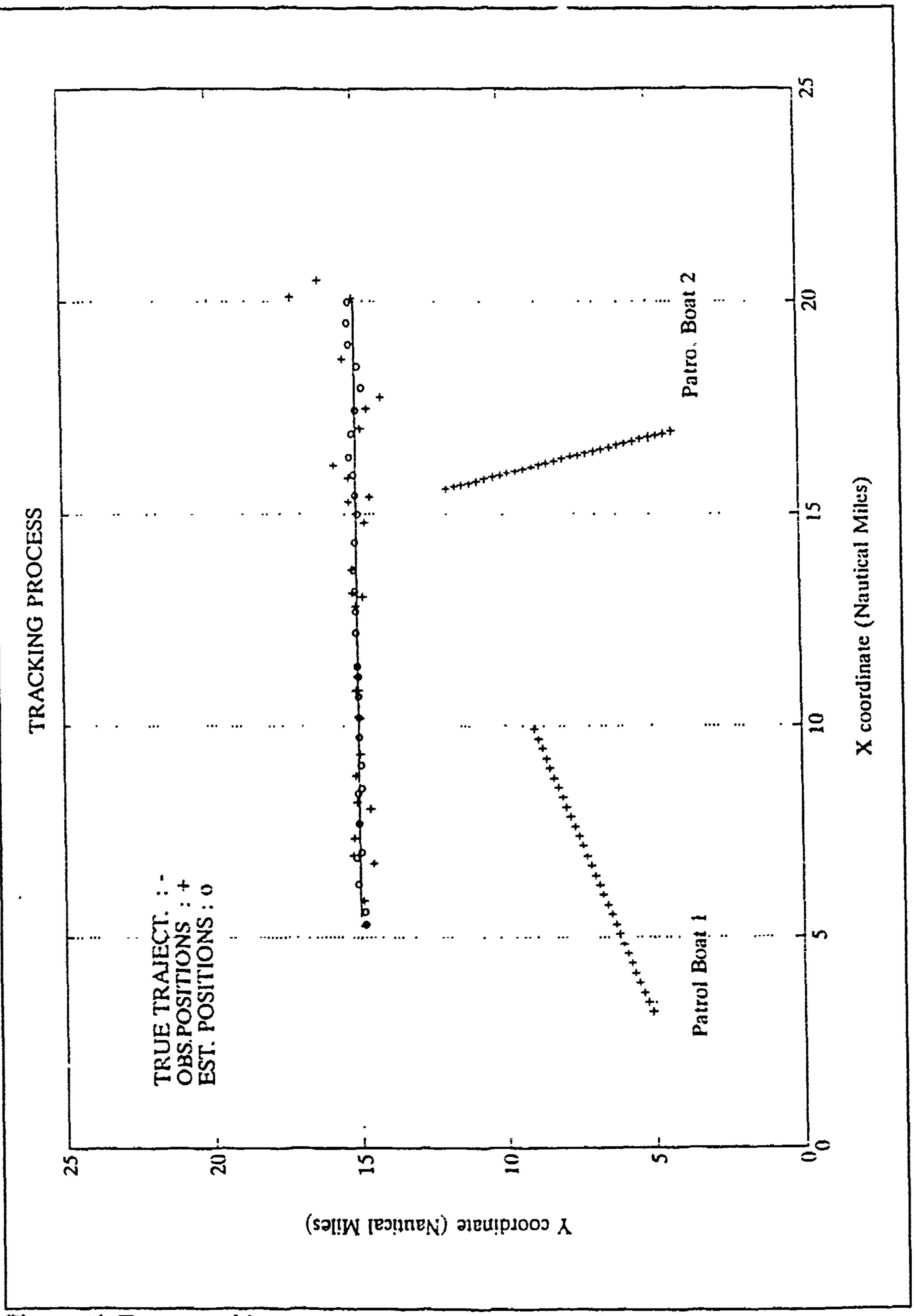

Figure 14. Target tracking

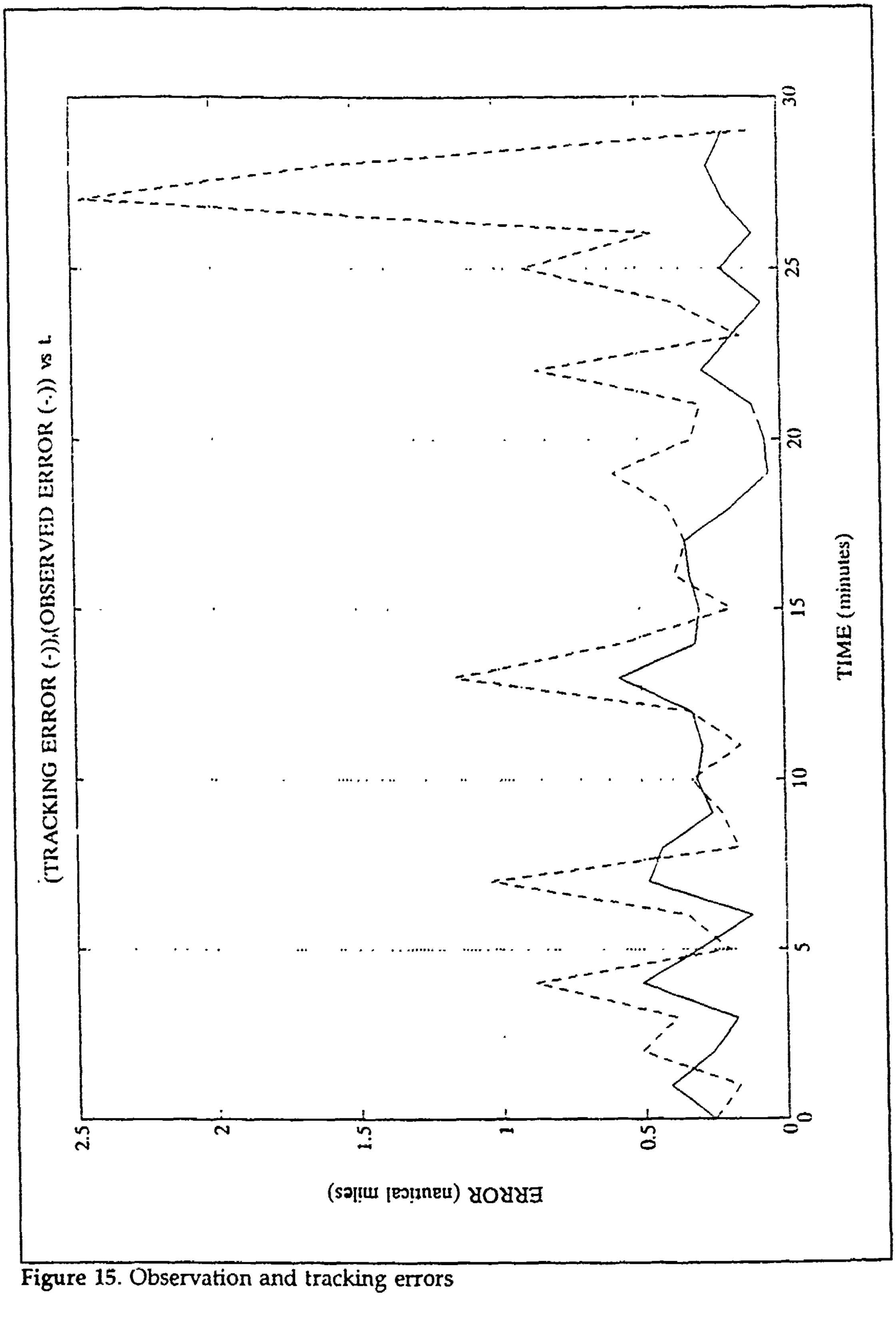

Figure 15. Observation and tracking errors

b. Scenario Nr 2.

In this case, the bearings provided by the ESM system with a DF accuracy of 7 degrees, will not allow the filter to use the maneuver gating in order to detect the target's maneuver. After the maneuver takes place, the tracking error increases to 1.35 nm diminishing to 0.85 at the end of the process. The results for this scenario are shown in Figures 16 and 17, and the output file TRKINFO containing the estimated target's data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.297562	14.839560	0.000000	0.000000
1	5.606289	14.889010	87.872260	3.000554
2	6.251655	15.076120	84.202790	13.723970
3	6.878047	15.129430	85.245830	21.320830
4	6.996462	14.959290	92.128600	17.180810
5	7.691809	15.012230	89.670380	23.334330
6	8.393325	15.057860	88.702640	27.468300
7	8.522616	14.936160	91.944220	23.643440
8	9.073889	14.943670	91.379620	25.277470
9	9.745502	14.987170	90.270650	27.624500
10	10.196570	14.979060	90.378960	27.544940
11	10.714290	14.987440	90.189420	28.005710
12	11.179040	14.995240	90.040020	27.990760
13	11.423670	15.014810	89.716520	26.488830
14	12.195530	15.038620	89.425810	28.582370
15	12.676740	15.133140	88.241260	28.624480
16	13.079730	15.296360	86.339990	28.256310
17	13.478650	15.497200	84.218650	27.967140
18	14.021290	15.657320	82.972950	28.445910
19	14.554630	15.811330	81.983630	28.826030
20	14.878760	16.117730	79.330950	28.344940
21	15.212270	16.431530	76.872040	28.021690
22	15.473110	16.844350	73.551480	27.615560
23	15.870110	17.104770	72.254400	27.633100
24	16.274770	17.358180	71.178310	27.681020
25	16.698180	17.576720	70.557170	27.745740
26	17.037490	18.017270	67.980550	27.910070
27	17.338860	18.635320	64.223680	28.363990
28	17.697270	19.110920	62.089860	28.751550
29	18.087560	19.504630	60.839600	29.052230

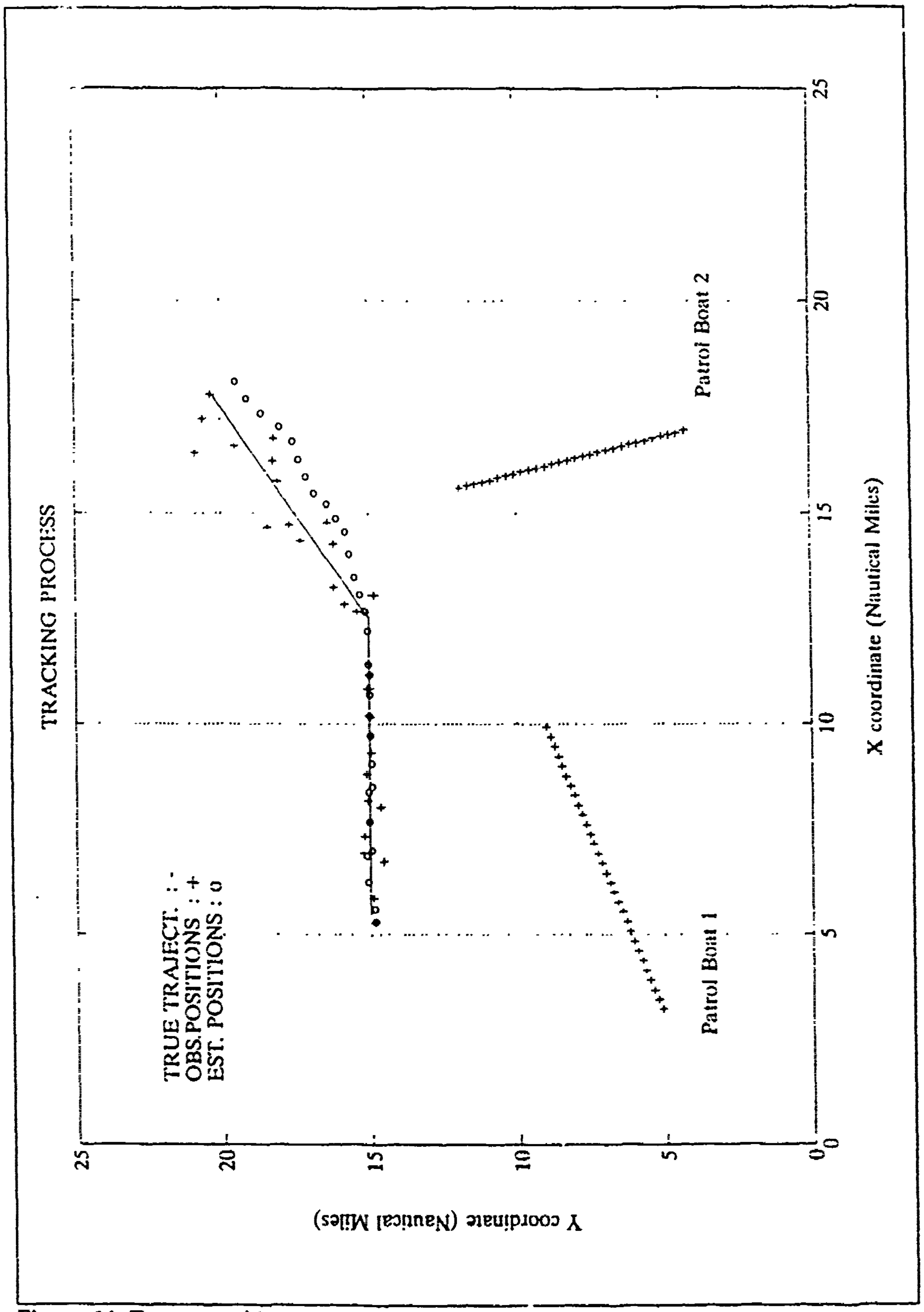

Figure 16. Target tracking

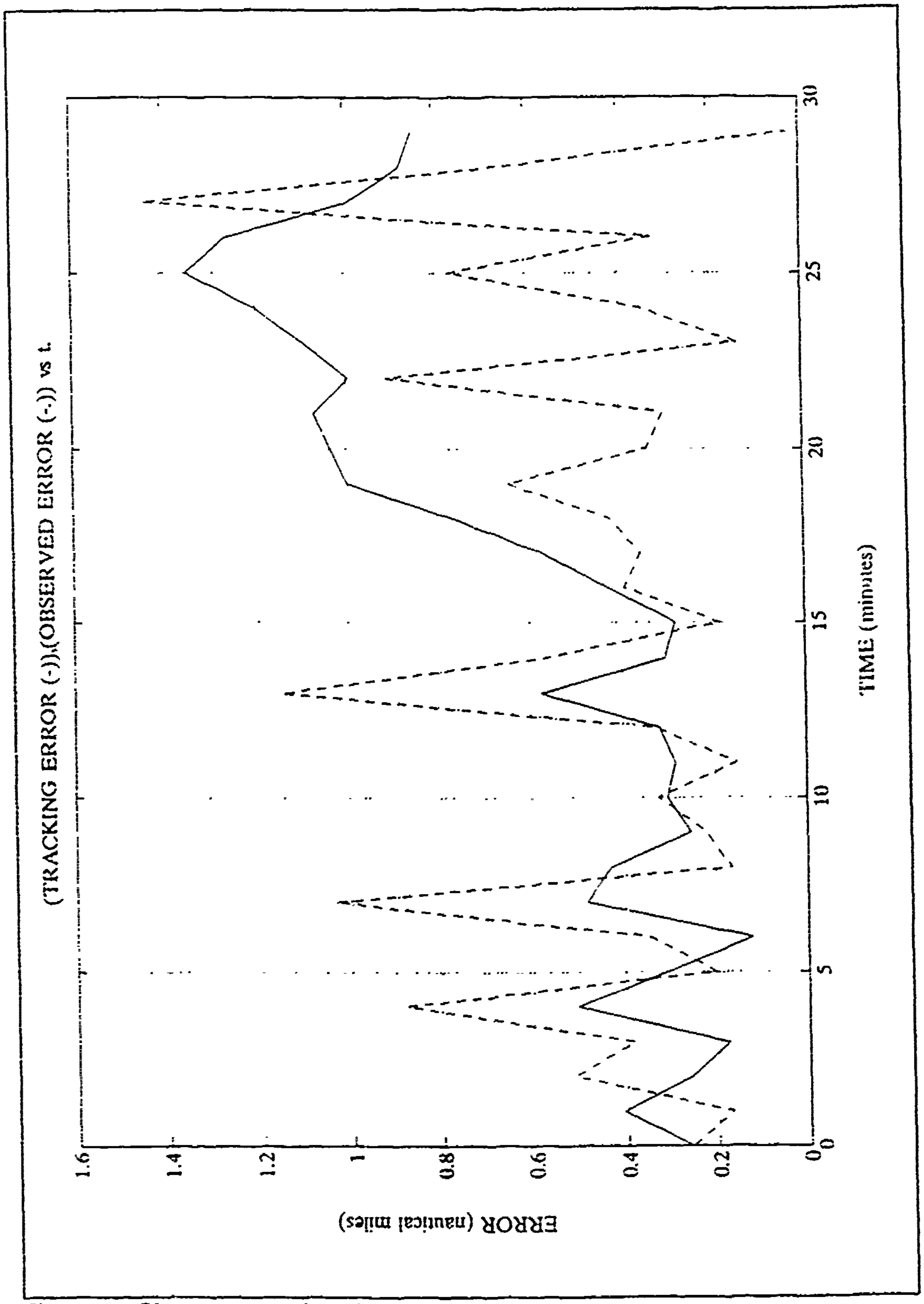

Figure 17. Observation and tracking errors

c. *Scenario Nr 3.*

The 90 degrees maneuver is detected by the filter's maneuver gating by evaluating eight (8) observations after the maneuver took place at 15 minutes. While the maneuver was not detected, the tracking error reaches a value of 1.92 nm which is later decreased to 0.95 nm. The results for this scenario are shown in Figures 18 and 19, and the output file TRKINFO containing the target's estimated data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.297562	14.839560	0.000000	0.000000
1	5.606289	14.889010	87.872260	3.000554
2	6.251655	15.076120	84.202790	13.723970
3	6.878047	15.129430	85.245830	21.320830
4	6.996462	14.959290	92.128600	17.180810
5	7.691809	15.012230	89.670380	23.334330
6	8.393325	15.057860	88.702640	27.468300
7	8.522616	14.936160	91.944220	23.643440
8	9.073889	14.943670	91.379620	25.277470
9	9.745502	14.987170	90.270650	27.624500
10	10.196570	14.979060	90.378960	27.544940
11	10.714290	14.987440	90.189420	28.005710
12	11.179040	14.995240	90.040020	27.990760
13	11.423670	15.014810	89.716520	26.488830
14	12.195530	15.038620	89.425810	28.582370
15	12.585460	15.160430	87.851670	28.087250
16	12.833990	15.378470	85.125050	26.935080
17	13.034070	15.654520	81.796400	25.789000
18	13.345960	15.922200	79.019260	25.423120
19	13.632530	16.215840	76.187030	25.068460
20	13.699830	16.628960	71.648550	24.019530
21	13.765090	17.047440	67.284070	23.231020
22	11.636970	19.155960	315.930000	49.322520
23	11.460090	19.848020	323.451900	45.293120
24	11.590990	20.522890	338.381500	40.542750
25	12.019920	21.131820	355.700000	38.295550
26	12.052590	21.684220	357.379900	37.427800
27	11.747040	22.235710	351.789100	36.885060
28	11.753030	22.768380	353.236700	36.140730
29	11.923520	23.291300	356.873500	35.452060

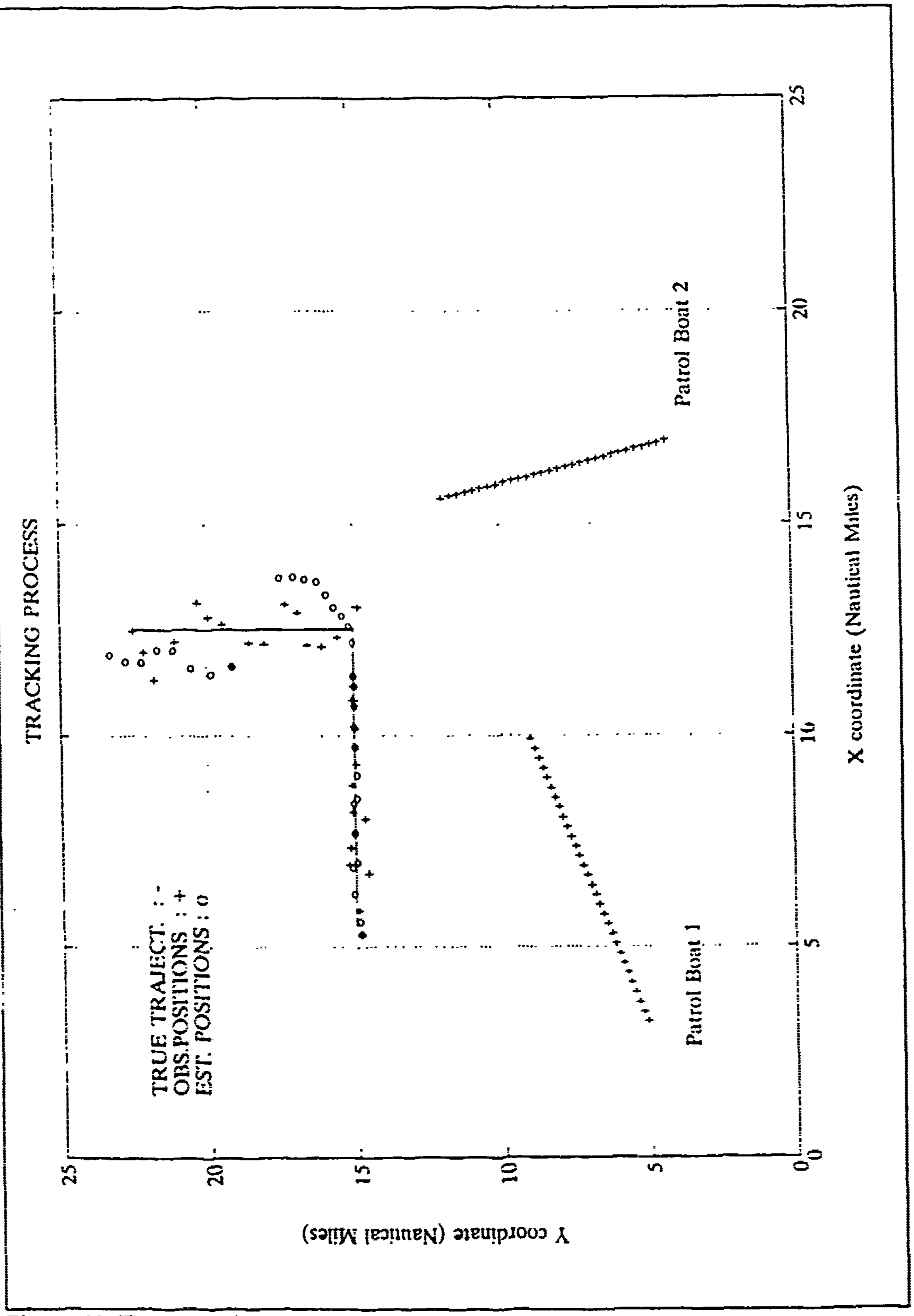

Figure 18. Target tracking

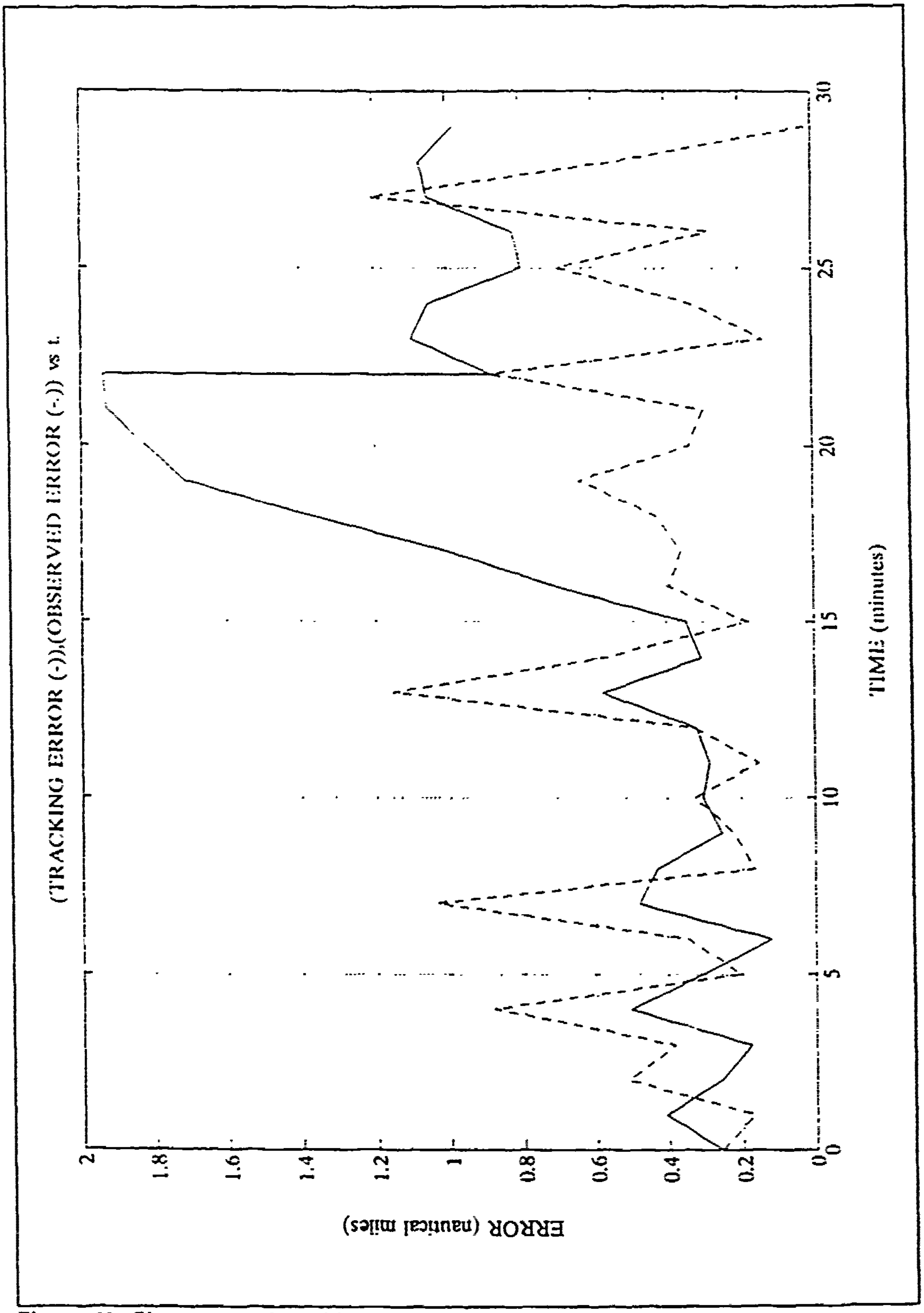

Figure 19. Observed and tracking errors

3. RDL-1BC ESM EQUIPMENT

a. *Scenario Nr 1.*

Although the DF accuracy for the current ESM system is very high and the observation error reaches a value of 4.2 nautical miles at 27 minutes, the filter is capable to follow the target's trajectory by keeping an average tracking error of 0.60 nm. The results for this scenario are shown in Figures 20 and 21, and the output file TRKINFO containing the target's estimated data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.160920	14.724770	0.000000	0.000000
1	5.468747	14.800420	86.873050	0.4379[illegible]7
2	6.084392	15.106420	83.274390	2.865865
3	6.548177	15.190230	84.770480	5.647289
4	6.525669	14.965950	92.359770	4.606804
5	7.084311	15.043260	88.849180	9.187359
6	7.743289	15.115360	87.670190	14.094480
7	7.809889	14.951200	91.927350	12.495720
8	8.364488	14.962200	91.087980	15.591290
9	9.107235	15.031990	89.456180	19.656760
10	9.575869	15.015540	89.833470	20.768670
11	10.138350	15.029510	89.635210	22.367990
12	10.624040	15.036990	89.568810	23.150570
13	10.769660	15.030270	89.698510	21.586090
14	11.718900	15.123080	88.474850	25.210810
15	12.280000	15.145620	88.364200	26.030660
16	12.761050	15.179520	88.108360	26.295410
17	13.272120	15.216480	87.856860	26.682810
18	14.057480	15.164190	88.628620	28.374460
19	14.851450	15.028410	90.027480	29.899680
20	15.298050	15.095960	89.409730	29.665790
21	15.776880	15.145780	89.010560	29.601390
22	16.156480	15.358590	87.234490	29.148330
23	16.755520	15.213640	88.651090	29.595090
24	17.341010	14.991460	90.502660	29.961640
25	17.899970	14.659020	92.976840	30.240700
26	18.395630	14.819460	91.609440	30.175070
27	18.913200	15.233950	88.527730	30.197700
28	19.425000	15.317550	88.033440	30.229680
29	19.914740	15.235900	88.716510	30.184120

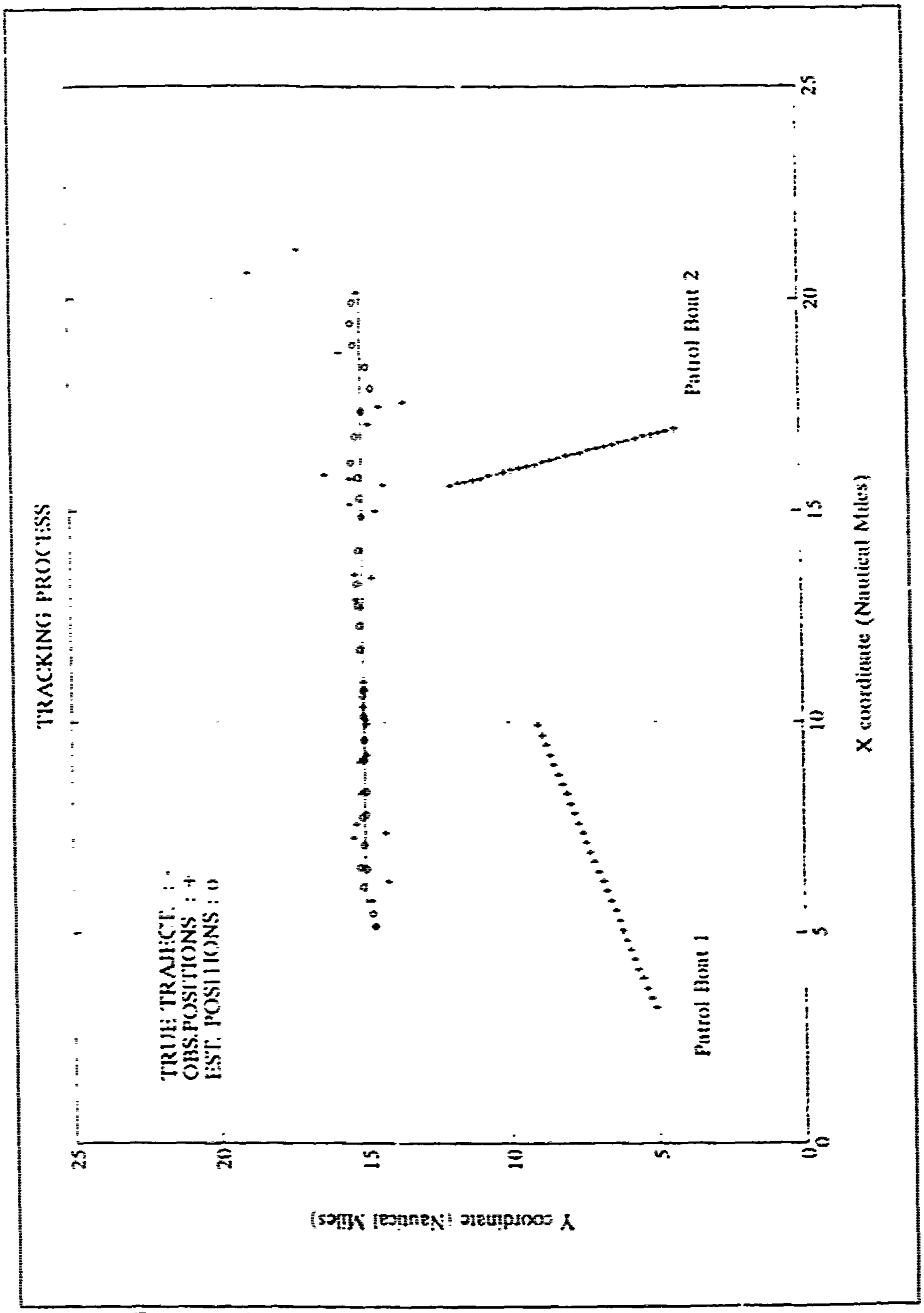

Figure 20. Target tracking

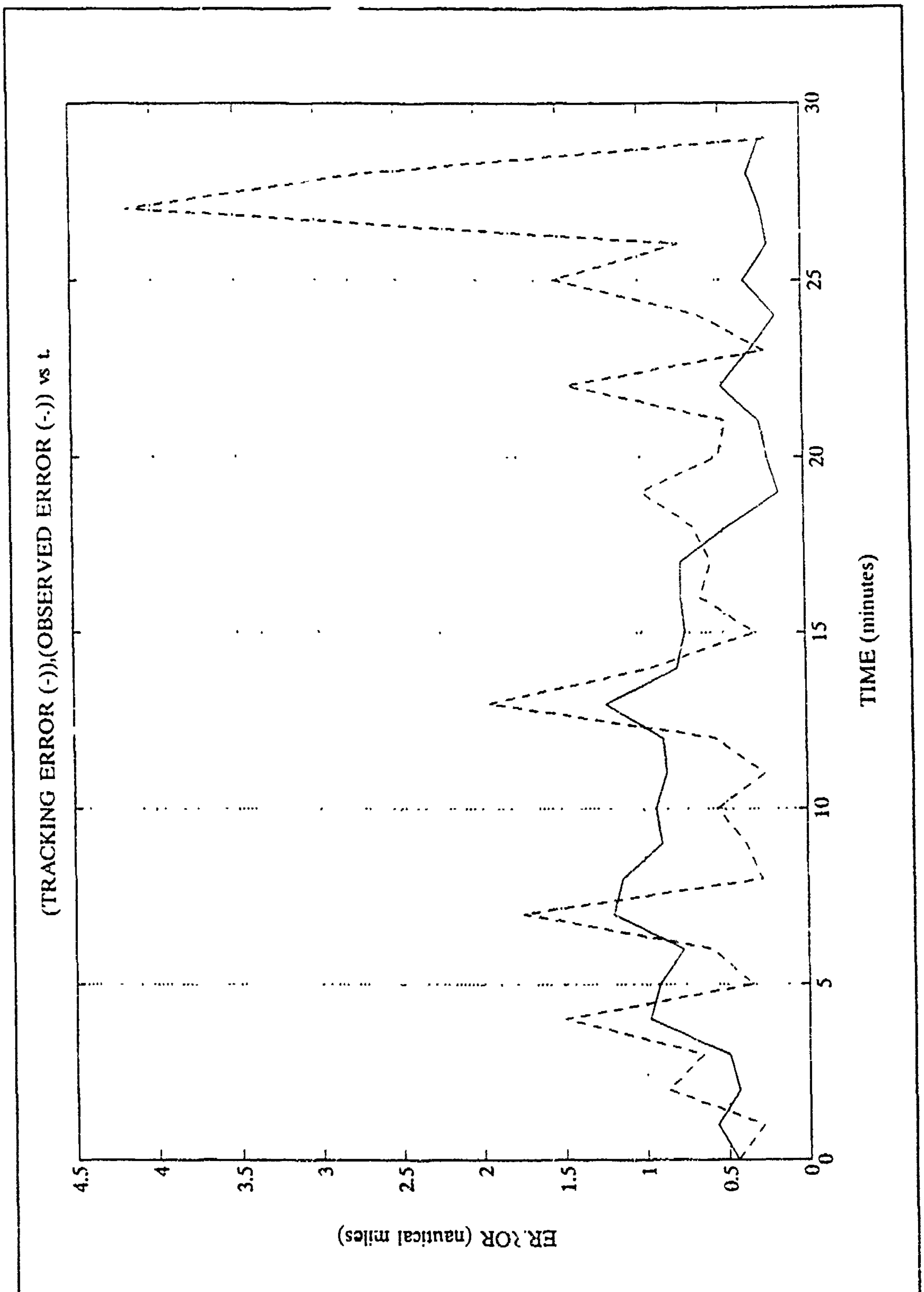

Figure 21. Observation and tracking errors

b. Scenario Nr 2.

The DF accuracy of 20 degrees for the current ESM system giving an observation error with high values such as 1.5, 1.7, 1.95 and 2.4 nautical miles, will drive the filter to fail in the use of the maneuver gating to detect the target change of course, with the tracking error increasing from 0.42 to 1.62 nautical miles at 25 minutes. The results for this scenario are shown in Figures 22 and 23, and the output file TRKINFO containing the ta[illegible] estimated data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.160920	14.724770	0.000000	0.000000
1	5.468747	14.800420	86.873050	0.437957
2	6.084392	15.106420	83.274390	2.865865
3	6.548177	15.190230	84.770480	5.647289
4	6.525669	14.965950	92.359770	4.606804
5	7.084311	15.043260	88.849180	9.187359
6	7.743289	15.115360	87.670190	14.094480
7	7.809889	14.951200	91.927350	12.495720
8	8.364488	14.962200	91.087980	15.591290
9	9.107235	15.031990	89.456180	19.656760
10	9.575869	15.015540	89.833470	20.768670
11	10.138350	15.029510	89.635210	22.367990
12	10.624040	15.036990	89.568810	23.150570
13	10.769660	15.030270	89.698510	21.586090
14	11.718900	15.123080	88.474850	25.210810
15	12.242700	15.212580	87.485190	25.829440
16	12.656120	15.362420	85.835110	25.783480
17	13.072130	15.549900	83.940970	25.797750
18	13.735540	15.686430	83.139880	27.027680
19	14.395680	15.785930	82.863380	28.071040
20	14.706770	16.076720	80.412510	27.546340
21	15.033500	16.370960	78.166890	27.193910
22	15.234080	16.799460	74.644880	26.545540
23	15.668290	17.002680	73.907650	26.678530
24	16.114670	17.180200	73.498730	26.816180
25	16.591350	17.288950	73.749410	26.973330
26	16.904000	17.716840	71.185680	26.903340
27	17.129320	18.385210	66.893610	26.929530
28	17.449740	18.827900	64.814590	27.026090
29	17.822280	19.143750	63.907220	27.105550

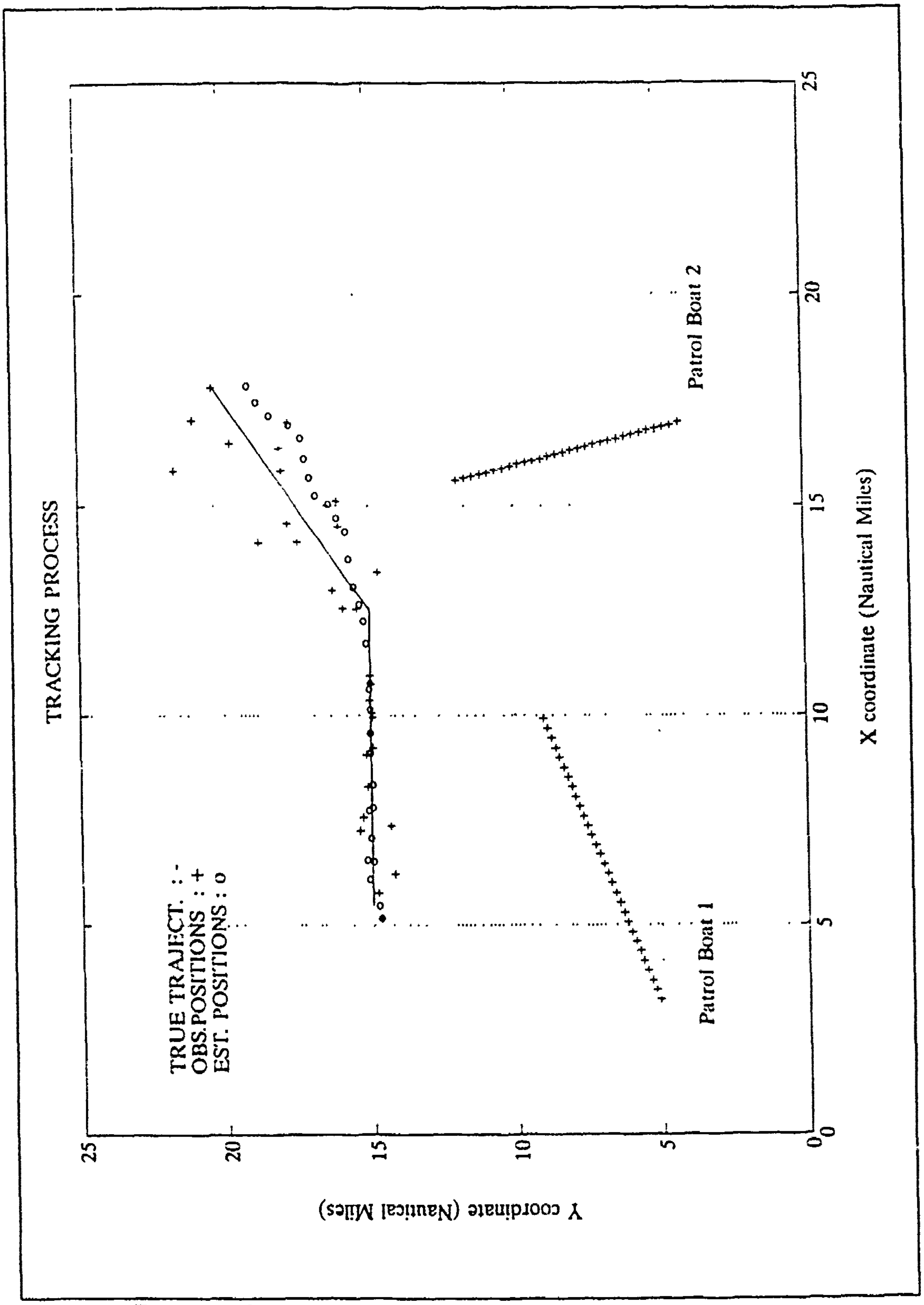

Figure 22. Target tracking

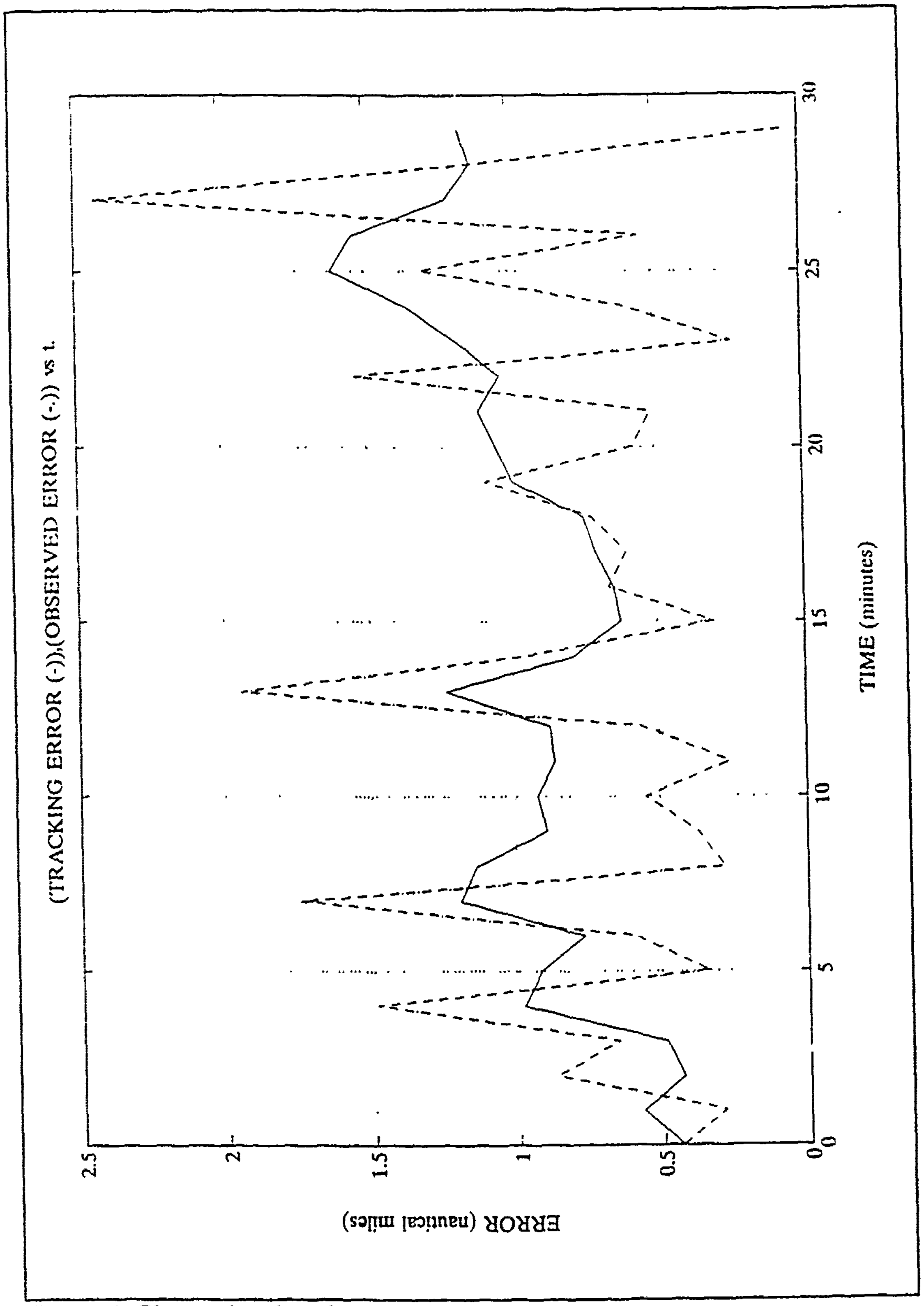

Figure 23. Observed and tracking errors

c. Scenario Nr 3.

In this case, the poor DF accuracy of the ESM system will not allow the filter to use the maneuver gating algorithm to detect the target's maneuver at 15 minutes, driving the tracking error from 0.42 to 2.78 nautical miles after the maneuver took place. The results of this scenario are shown in Figures 24 and 25, and the output file TRKINFO containing the target's estimated data is presented below.

TIME(min)	X POS(nm)	Y POS(nm)	HDG(deg)	SPD(knots)
0	5.1[illegible]0920	14.724770	0.000000	0.000000
1	5.468747	14.800420	86.873050	0.437957
2	6.084392	15.106420	83.274390	2.865865
3	6.548177	15.190230	84.770480	5.647289
4	6.525669	14.965950	92.359770	4.606804
5	7.084311	15.043260	88.849180	9.187359
6	7.743289	15.115360	87.670190	14.094480
7	7.809889	14.951200	91.927350	12.495720
8	8.364488	14.962200	91.087980	15.591290
9	9.107235	15.031990	89.456180	19.656760
10	9.575869	15.015540	89.833470	20.768670
11	10.138350	15.029510	89.635210	22.367990
12	10.624040	15.036990	89.568810	23.150570
13	10.769660	15.030270	89.698510	21.586090
14	11.718900	15.123080	88.474850	25.210810
15	12.156560	15.227600	87.244830	25.334480
16	12.420440	15.413070	84.993490	24.533650
17	12.639780	15.653360	82.140310	23.684380
18	13.069830	15.881950	80.033790	24.024470
19	13.479370	16.120920	78.075900	24.264370
20	13.533460	16.506220	73.861470	23.103980
21	13.591340	16.895810	69.760210	22.210330
22	13.504770	17.352460	64.496540	21.081140
23	13.660610	17.700180	61.726940	20.874680
24	13.833940	18.046500	59.314390	20.786[illegible]00
25	14.053920	18.374690	57.546620	20.849960
26	14.062400	18.819900	53.822770	20.537330
27	13.918370	19.336330	48.872710	20.080590
28	13.903470	19.753940	45.757290	19.871890
29	13.969240	20.130380	43.677430	19.818920

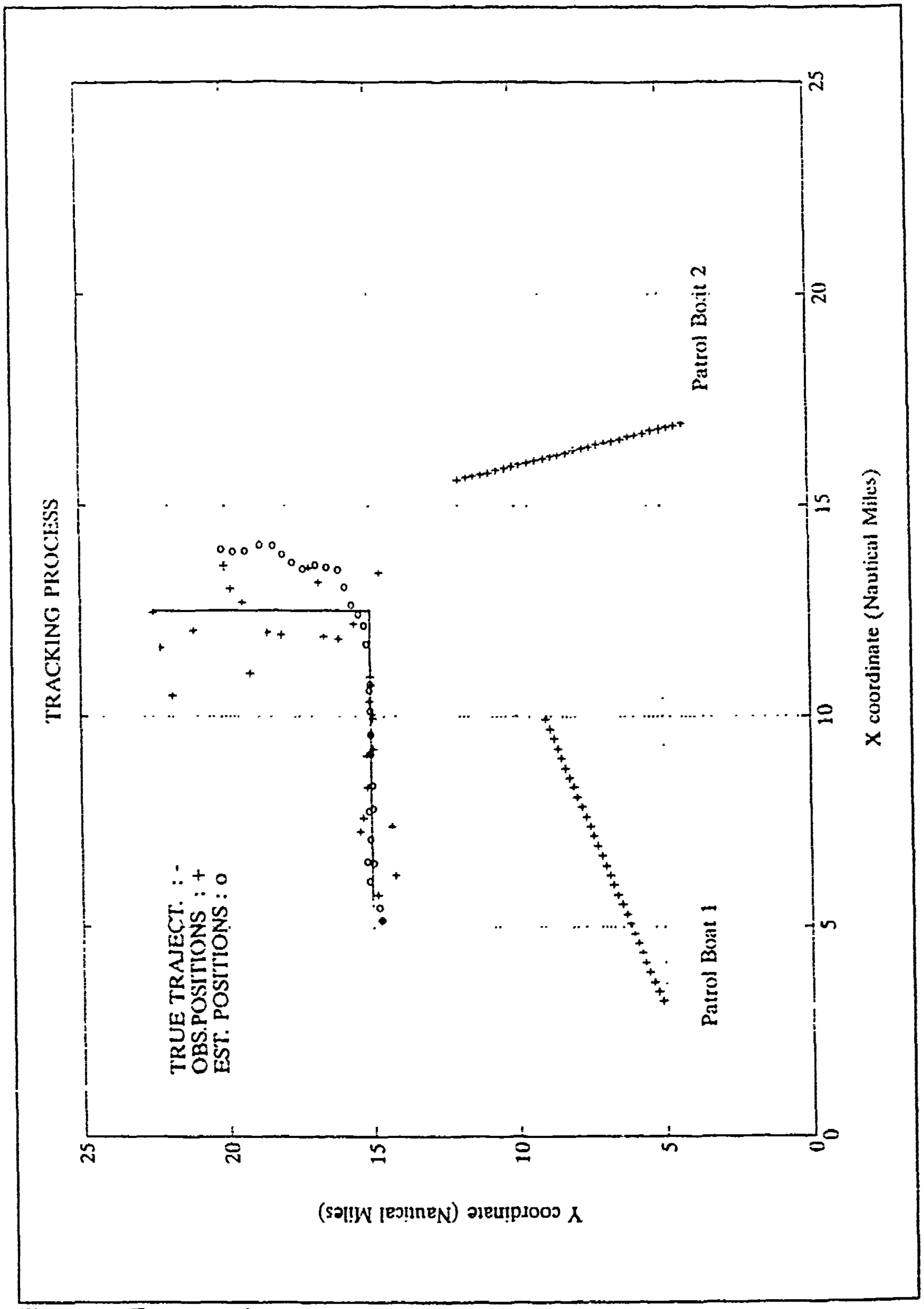

Figure 24. Target tracking

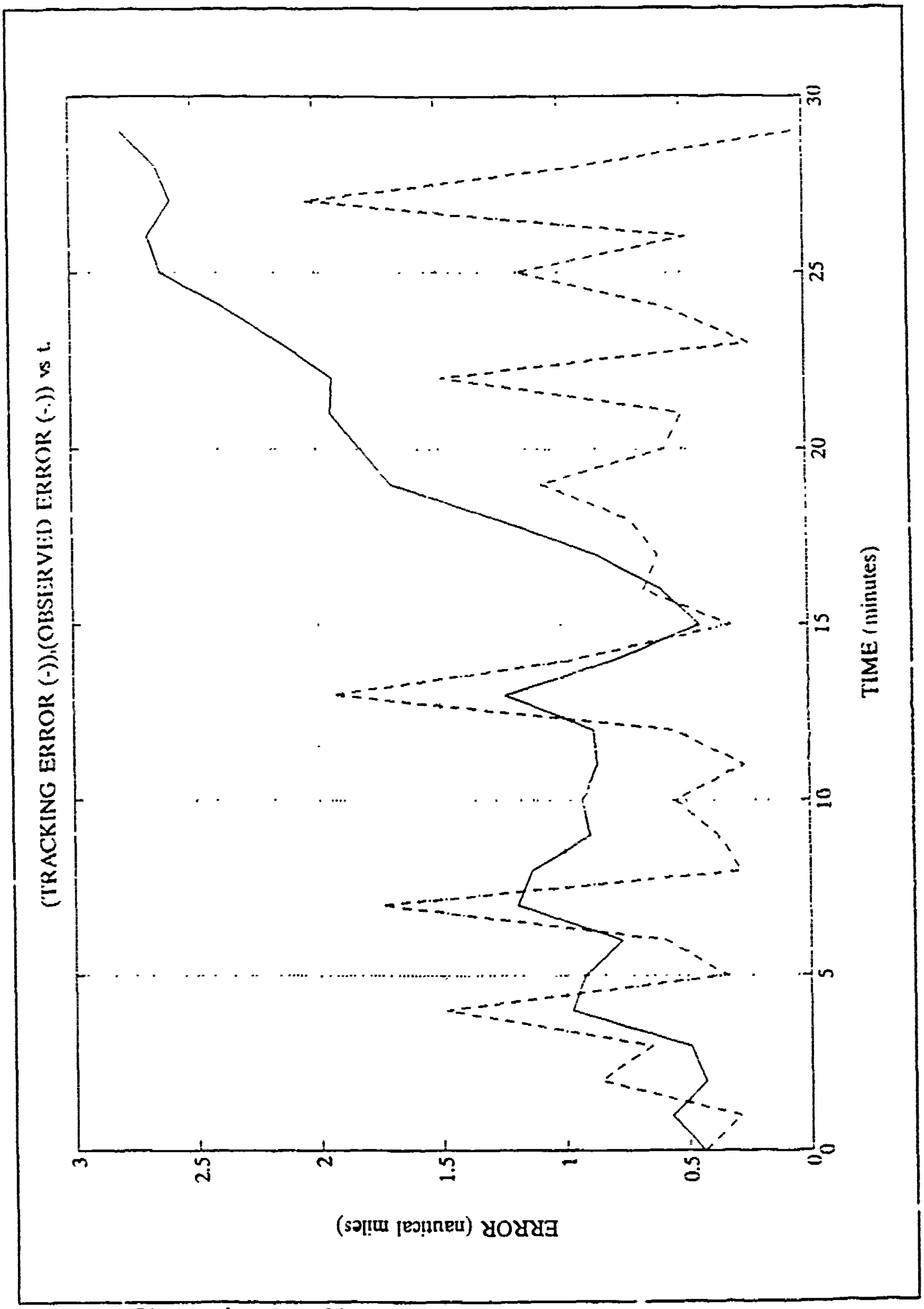

Figure 25. Observed and tracking errors

VII. CONCLUSIONS

The results obtained from the different situations involving the DF accuracy of ESM systems for the tracking process and maneuver detection of the boat, represents how important the information provided by the ESM system is. Its accuracy allows the Kalman filter to process the observations with a high probability of the estimated data being close to the real trajectory of the target. In the event of high DF accuracy, the received signals involve a high spread in bearing indication that will affect the performance of the filter which would fail in the detection of the target's maneuver as observed in the results obtained for the ELETTRONICA EW EQUIPMENT and the RDL-1BC ESW EQUIPMENT.

The presented scenarios were analyzed in detail by running the programs with theoretical DF accuracies from 1 to 20 degrees, to find the exact DF value where the maneuver gating no longer works. This value was determined as 4.6 degrees for the second scenario which involves a change of course of 45 degrees and the maneuver gating stopped working at 9.6 degrees for the third scenario which involves a 90 degrees maneuver.

The improvement of the tracking process as the DF accuracy increases, was also calculated from these simulations and the results are presented in Figure 26, where the degradation of the process is seen as the error of the ESM system increases.

The atmospheric noise and other propagation factors that affect the received signals on the ESM equipment were not considered due to the close range between the target and the patrol boats for the different scenarios. However, this can be a consideration for further

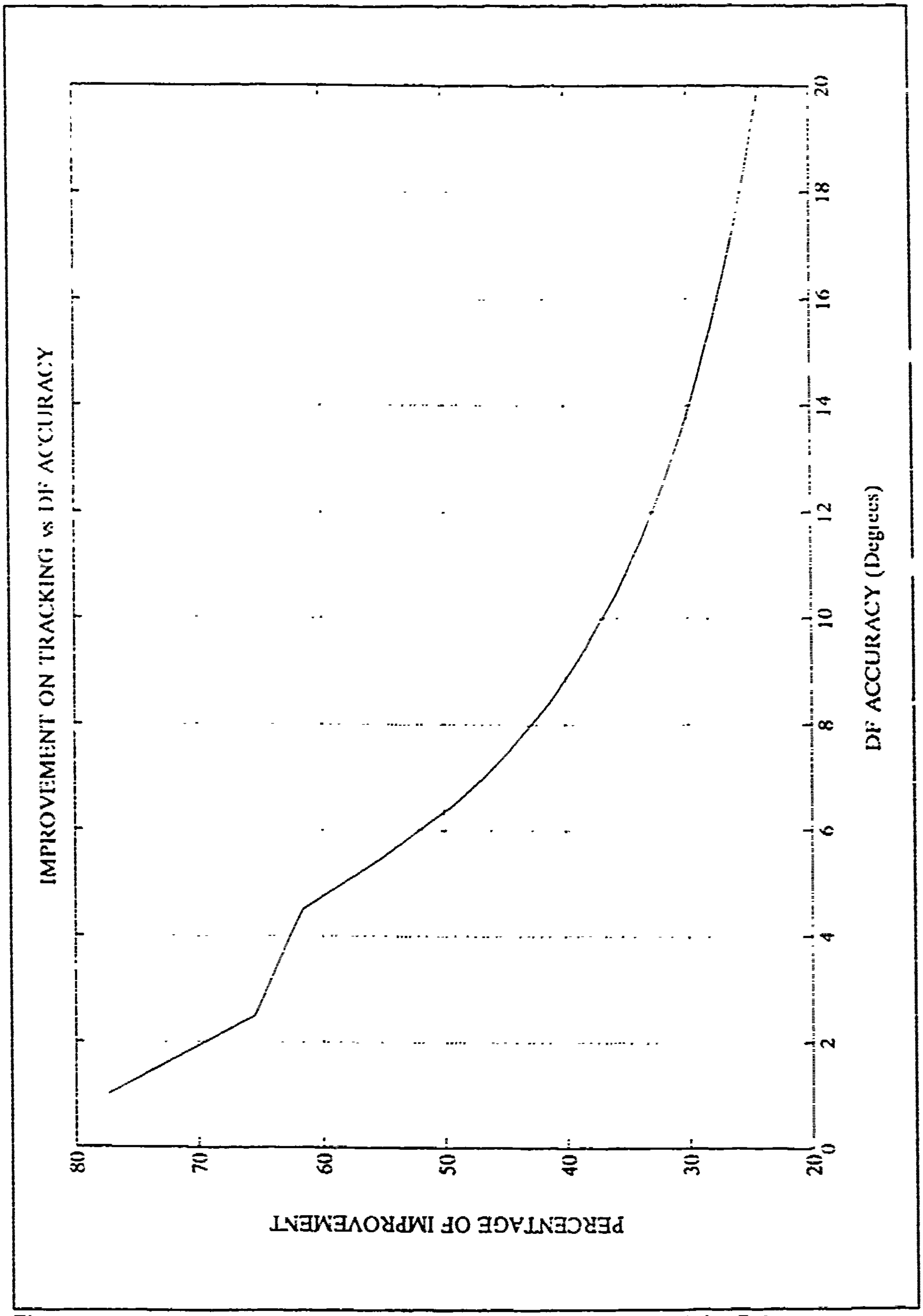

Figure 26. Improvement of the tracking process as a function of the DF accuracy

analysis where the DF accuracy of the selected systems can be confronted in a long distance or OTH tracking situation.

APPENDIX A. PROGRAM TRAKDATA

```
C                          ***TRACKDATA***
C ***************************************************************************
C THIS PROGRAM COMPUTES THE TRUE TARGET AND SENSOR POSITIONS AND
C CORRESPONDING TARGET BEARINGS FOR USED DESIGNATED OBSERVATION
C TIMES.
C ***************************************************************************

C VARIABLE DECLARATIONS

      REAL*4  XT(4,1),XS1(4,1),PHI(4,4),SPDS1,HDGS1,SPDS2,HDGS2
      REAL*4  DT,HDGT,SPDT,XS2(4,1),TEMP1(4,1),CASE,XDIFF1,YDIFF1
      REAL*4  XDIFF2,YDIFF2,NOISE,DTOR,RTOD,BRG1,BRG2
      REAL*4  MANXT(4,1),MANHDGT,MANSPDT
      INTEGER TIME,TIMEM1,MANTIME


C DEFINE THE INPUT AND OUTPUT FILES

      OPEN(UNIT=3,FILE='NOISEFIL',STATUS='OLD')
      OPEN(UNIT=4,FILE='TRKDATA',STATUS='NEW')

      WRITE(*,*)'ENTER A NEGATIVE NUMBER FOR NOISELESS CASE;'
      WRITE(*,*)'POSITIVE FOR NOISY CASE'
      READ(*,*)CASE

      TIMEM1=0
      RTOD=57.29577951
      DTOR=0.017453293

C     INPUT THE TARGET TRACK PARAMETERS

      WRITE(*,*)'INPUT DESIRED INITIAL X POSITION OF TARGET'
      READ(*,*) XT(1,1)
      WRITE(*,*)'INPUT DESIRED INITIAL  Y POSITION OF TARGET'
      READ(*,*) XT(3,1)
      WRITE(*,*)'INPUT DESIRED TARGET COURSE IN DEGREES'
      READ(*,*)HDGT
      WRITE(*,*)'INPUT DESIRED TARGET SPEED IN KNOTS'
      READ(*,*)SPDT
```

```
      XT(2,1)=(SPDT/60)*SIN(HDGT*DTOR)
      XT(4,1)=(SPDT/60)*COS(HDGT*DTOR)

C     INPUT THE SENSOR TRACK PARAMETERS

      WRITE(*,*)'FOR SENSOR 1:'
      WRITE(*,*)'INPUT DESIRED INITIAL X POSITION'
      READ(*,*)XS1(1,1)
      WRITE(*,*)'INPUT DESIRED INITIAL Y POSITION'
      READ(*,*)XS1(3,1)
      WRITE(*,*)'INPUT DESIRED COURSE IN DEGREES'
      READ(*,*)HDGS1
      WRITE(*,*)'INPUT DESIRED SPEED IN KNOTS'
      READ(*,*)SPDS1

      XS1(2,1)=(SPDS1/60)*SIN(HDGS1*DTOR)
      XS1(4,1)=(SPDS1/60)*COS(HDGS1*DTOR)

      WRITE(*,*)'FOR SENSOR 2:'
      WRITE(*,*)'INPUT DESIRED INITIAL X POSITION'
      READ(*,*)XS2(1,1)
      WRITE(*,*)'INPUT DESIRED INITIAL Y POSITION'
      READ(*,*)XS2(3,1)
      WRITE(*,*)'INPUT DESIRED COURSE IN DEGREES'
      READ(*,*)HDGS2
      WRITE(*,*)'INPUT DESIRED SPEED IN KNOTS'
      READ(*,*)SPDS2

      XS2(2,1)=(SPDS2/60)*SIN(HDGS2*DTOR)
      XS2(4,1)=(SPDS2/60)*COS(HDGS2*DTOR)

C     INPUT THE TRUE TRACK UPDATES

300   WRITE(*,*)'INPUT TIME INTERVAL OF CALCULATIONS'
      WRITE(*,*)'(NEG.FOR END OF PROBLEM)'
      READ(*,*)TIME
      WRITE(*,*)'ENTER TIME FOR TARGET MANEUVER IF SO'
      WRITE(*,*)'IF NOT MANEUVER ENTER 0'
      READ(*,*)MANTIME

      IF (MANTIME.EQ.0) GOTO 400

      IF (MANTIME.NE.0) THEN
         WRITE(*,*)'INPUT MANEUVERING TARGET COURSE IN DEGREES'
         READ(*,*)MANHDGT
         WRITE(*,*)'INPUT MANEUVERING TARGET SPEED IN KNOTS'
         READ(*,*)MANSPDT
```

```
      MANXT(2,1)=(MANSPDT/60)*SIN(MANHDGT*DTOR)
      MANXT(4,1)=(MANSPDT/60)*COS(MANHDGT*DTOR)
    ENDIF

C  UPDATE TARGET AND SENSOR STATES TO MEASUREMENT TIME

400   DT=TIME-TIMEM1

C  COMPUTE PHI MATRIX
    PHI(1,1)=1.0
    PHI(1,2)=DT
    PHI(1,3)=0.0
    PHI(1,4)=0.0
    PHI(2,1)=0.0
    PHI(2,2)=1.0
    PHI(2,3)=0.0
    PHI(2,4)=0.0
    PHI(3,1)=0.0
    PHI(3,2)=0.0
    PHI(3,3)=1.0
    PHI(3,4)=DT
    PHI(4,1)=0.0
    PHI(4,2)=0.0
    PHI(4,3)=0.0
    PHI(4,4)=1.0

C  INITIATE THE PROCESS OF TRACK DATA GENERATION

    DO 310 I=1,30

    TIME = I*DT-1

    IF (MANTIME.EQ.0) GOTO 600
    IF (TIME.EQ.MANTIME)  THEN
      XT(2,1)=MANXT(2,1)
      XT(4,1)=MANXT(4,1)
    ENDIF

C  UPDATE TARGET STATES

600   CALL MATMUL(PHI,XT,4,4,1,TEMP1)
    DO 700 K=1,4
        XT(K,1)=TEMP1(K,1)
700   CONTINUE
```

```
C  UPDATE SENSOR STATES

      CALL MATMUL(PHI,XS1,4,4,1,TEMP1)
      DO 710 L=1,4
           XS1(L,1)=TEMP1(L,1)

710   CONTINUE

      CALL MATMUL(PHI,XS2,4,4,1,TEMP1)
      DO 720 M=1,4
           XS2(M,1)=TEMP1(M,1)

720   CONTINUE

      XDIFF1=XT(1,1)-XS1(1,1)
      YDIFF1=XT(3,1)-XS1(3,1)

      XDIFF2=XT(1,1)-XS2(1,1)
      YDIFF2=XT(3,1)-XS2(3,1)

      READ(3,*) NOISE

      IF (CASE.GE.0.0) GOTO 450
      NOISE=0.0

450   BRG1=RTOD*ATAN2(XDIFF1,YDIFF1)+NOISE
        IF (BRG1.LT.0.0) BRG1=BRG1+360
      BRG2=RTOD*ATAN2(XDIFF2,YDIFF2)+NOISE
        IF (BRG2.LT.0.0) BRG2=BRG2+360

      WRITE(4,500)TIME,XT(1,1),XT(3,1),XS1(1,1),XS1(3,1),
     *         BRG1,XS2(1,1),XS2(3,1),BRG2
500   FORMAT(I4,8F9.4)

310   CONTINUE

900   STOP

      END
```

```
      SUBROUTINE MATMUL(A,B,L,M,N,C)
C ************************************************************************
C     THIS ROUTINE MULTIPLIES TWO MATRICES TOGETHER
C          C(L,N) = A(L,M) * B(M,N)
C ************************************************************************
C              DIMENSIONS AND DECLARATIONS

      REAL*4 A(L,M),B(M,N),C(L,N)

      DO 10 I=1,L
      DO 10 J=1,N
       C(I,J)=0.0
10    CONTINUE

         DO 100 I= 1,L
         DO 100 J= 1,N
         DO 100 K= 1,M
      C(I,J) = C(I,J) + A(I,K)*B(K,J)
100   CONTINUE

      RETURN
      END
```

APPENDIX B. PROGRAM SHIPTRACK

```
C                              ***SHIPTRACK***
C**********************************************************************************
C THIS PROGRAM EMPLOYS AN ADAPTIVE EXTENDED KALMAN FILTER TO
C TRACK A MANEUVERING BOAT TARGET USING BEARINGS-ONLY RADIO
C DIRECTION-FINDING MEASUREMENTS FROM TWO INDEPENDENT SENSORS
C**********************************************************************************

C VARIABLE DEFINITIONS

C   BRG      = MEASURED TARGET BEARING IN RADIANS
C   BRKKM1   = PREDICTED TARGET BEARING MEASUREMENT IN RADIANS
C                BRG(k/k-1)
C   DBRG     = MEASURED TARGET BEARING IN DEGREES
C   DEL      = STATE NOISE COEFFICIENT MATRIX
C   DFACC    = DF ACCURACY OF ESM SYSTEM
C   DT       = TIME DELAY BETWEEN OBSERVATIONS,  T(k)-T(k1)
C   DTOR     = DEGREE TO RADIAN CONVERSION FACTOR
C   E        = OBSERVATION RESIDUAL,  BRG-ATAN(XDIFF/YDIFF)
C   E1,E2    = MEASUREMENT RESIDUAL,  Z(k)-H*X(k/k-1)
C   E1M1,E2M1 = MEASUREMENT RESIDUAL AT PREVIOUS OBSERVATION
C   E1M2,E2M2 = MEASUREMENT RESIDUAL TWO PREVIOUS OBSERVATIONS
C   FAC1     = RECIPROCAL OF VARIANCE OF RESIDUALS (VARE)
C   G        = KALMAN GAIN VECTOR
C   GATE1    = 1.5*STANDARD DEVIATION OF RESIDUAL PROCESS,
C              USED AS A GATE IN MANEUVER DETECTION
C   H        = MEASUREMENT MATRIX
C   HDG      = ESTIMATED TARGET HEADING IN DEGREES
C   HT       = TRANSPOSE OF H
C   I        = COUNTER
C   IMAT     = 4 X 4 IDENTITY MATRIX
C   J        = COUNTER
C   K        = ITERATION INTERVAL
C   L        = SENSORS,   L = 1 and 2
C   LPKK     = STATE COVARIANCE MATRIX AFTER PREVIOUS
C                   OBSERVATIONS
C   LPKKM1   = A PRIORI STATE COVARIANCE ESTIMATE
C   LXKK     = STATE ESTIMATE AFTER PREVIOUS OBSERVATIONS
C   LXKKM1   = A PRIORI STATE ESTIMATE
C   M1,M2    = AVERAGE OF RESIDUALS OVER LAST THREE
C                   OBSERVATIONS
C   OBSERR   = OBSERVATION ERROR
```

```
C    PHI       = DISCRETE-TIME STATE TRANSITION MATRIX
C    PHIT      = TRANSPOSE OF PHI
C    PI        = 3.141592654
C    PKK       = ESTIMATION ERROR COVARIANCE MATRIX, P(k/k)
C    PKKM1     = PREDICTED ESTIMATION ERROR COVARIANCE MATRIX,
C                    P(k/k-1)
C    Q         = STATE EXCITATION COVARIANCE MATRIX
C    R         = MEASUREMENT NOISE COVARIANCE
C    RANGE     = DISTANCE FROM SENSOR TO A PRIORI TARGET
C                    POSITION
C    RTOD      = RADIAN TO DEGREE CONVERSION FACTOR
C    SPD       = ESTIMATED TARGET SPEED IN KNOTS
C    TEMP      = TEMPORARY STORAGE MATRICES USED IN MATRIX
C                    OPERATIONS
C    TIME      = ACTUAL OBSERVATION TIME
C    TIMEM1    = TIME COUNTER
C    TRKERR    = TRUE TRACKING ERROR
C    VARE      = VARIANCE OF RESIDUALS PROCESS
C    XDIFF     = DISTANCE IN X DIRECTION FROM SENSOR TO A PRIORI
C                    TARGET POSITION
C    XKK       = ESTIMATED TARGET STATE VECTOR, X(k/k)
C    XKKM1     = PREDICTED TARGET STATE VECTOR, X(k/k-1)
C    XPOS      = ESTIMATED TARGET POSITION IN X DIRECTION
C    XS        = POSITION OF SENSOR IN X DIRECTION
C    XT        = TRUE TARGET POSITION IN X DIRECTION
C    YDIFF     = DISTANCE IN Y DIRECTION FROM SENSOR TO A PRIORI
C                    TARGET POSITION
C    YPOS      = ESTIMATED TARGET POSITION IN Y DIRECTION
C    YS        = POSITION OF SENSOR IN Y DIRECTION
C    YT        = TRUE TARGET POSITION IN Y DIRECTION
C    ZX        = OBSERVED POSITION IN X DIRECTION
C    ZY        = OBSERVED POSITION IN Y DIRECTION

C VARIABLE DECLARATIONS

      REAL XKK(4,1),XKKM1(4,1),PHI(4,4),DTOR
      REAL H(1,4),G(4,1),TEMP1(1,4),TEMP2(1,1),TEMP3(4,1)
      REAL TEMP4(4,4),TEMP5(4,4),PKK(4,4),PKKM1(4,4),HT(4,1)
      REAL LXKK(4,1),LPKK(4,4),XS(2),YS(2),DBRG(2),BRG(2)
      REAL TEMP6(4,4),TEMP7(4,4),PHIT(4,4),IMAT(4,4)
      REAL XT,YT,GATE1(2),DT,XDIFF,YDIFF,RANGE,Q(4,4)
      REAL XS1,YS1,BRG1,BRKKM1,E(2),VARE(2),FAC1
      REAL R,DFACC,RTOD,YS2,BRG2,ZX,ZY,M1,E1,E1M1,E1M2
      REAL LPKKM1(4,4),TRKERR,M2,E2,E2M1,E2M2,G11,G13,G21,G23
      REAL LXKKM1(4,1),ZXM1,ZYM1,OBSERR,DEL(4,2)
      INTEGER TIME,TIMEM1
```

```
C OPEN OUTPUT DATA FILES

      OPEN(UNIT=2,FILE='OUTDATA',STATUS='NEW')
      OPEN(UN:IT=3,FILE='TRKDATA',STATUS='OLD')
      OPEN(UNIT=4,FILE='ERRDATA',STATUS='NEW')
      OPEN(UNIT=5,FILE='TRKINFO',STATUS='NEW')

C RADIAN/DEGREE CONVERSION FACTORS

      RTOD=57.29577951
      DTOR=0.01745293

C COMPUTE 4X4 IDENTITY MATRIX

      DO 10 I=1,4
      DO 10 J=1,4
      IF (I.EQ.J) THEN
           IMAT(I,J)=1.0
      ELSE
           IMAT(I,J)=0.0
      ENDIF
10    CONTINUE

C INITIALIZE TIME COUNTER
      TIMEM1=0

C INITIALIZE COUNTER FOR MANEUVER GATE
      E1M1=0.0
      E1M2=0.0
      E2M1=0.0
      E2M2=0.0

C COMPUTE BEARING MEASUREMENT COVARIANCE
C     BEARING ERROR STANDARD DEVIATION = DF ACCURACY OF SYSTEM
      WRITE(*,*)'ENTER DF ACCURACY OF ESM SYSTEM'
      READ(*,*)DFACC
      R=(DFACC*DTOR)**2


C READ IN OBSERVATION PACKET (TIME, # OF SENSORS)
C     DT=TIME(k)-TIME(k-1)

20    READ(3,30,END=240)TIME,XT,YT,XS(1),YS(1),DBRG(1),
     *          XS(2),YS(2),DBRG(2)
30    FORMAT(I4,8F9.4)
```

```
      DO 40 L=1,2
           IF (DBRG(L).GT.180.0) DBRG(L)=DBRG(L)-360
      BRG(L)=DBRG(L)*DTOR

40    CONTINUE

      IF (TIME.LT.0) GOTO 240

      DT=TIME-TIMEM1

C CALCULATE THE PHI MATRIX

      CALL FINDPHI(PHI,DT)

      XS1=XS(1)
      YS1=YS(1)
      XS2=XS(2)
      YS2=YS(2)
      BRG1=BRG(1)
      BRG2=BRG(2)

C COMPUTE THE TARGET POSITION FROM BEARING MEASUREMENTS
      CALL MP(XS1,YS1,XS2,YS2,BRG1,BRG2,ZX,ZY)

      IF(TIME.EQ.0) THEN
          CALL INIT(XS1,YS1,XS2,YS2,BRG1,BRG2,XKK,PKK)

C          WRITE(*,*)'X(0/0,0):'
          DO 50 I=1,4
             LXKK(I,1)=XKK(I,1)
C              WRITE(*,*)XKK(I,1)
50         CONTINUE

C          WRITE(*,*)'P(0/0.0):'
          DO 70 I=1,4
           DO 70 J=1,4
             LPKK(I,J)=PKK(I,J)
C               WRITE(*,60)PKK(I,J)
60             FORMAT(4F14.4)
70         CONTINUE
      ENDIF

C PROJECT AHEAD THE STATE ESTIMATE
C     X(k+1/k) = PHI * X(k/k)
      CALL MATMUL(PHI,XKK,4,4,1,XKKM1)
```

```
C     WRITE(*,*)'X(',TIME,'/',TIMEM1,',0):'
      DO 80 I=1,4
C       WRITE(*,*)'XKKM1(I,1)
        LXKKM1(I,1)=XKKM1(I,1)

80    CONTINUE

C PROJECT AHEAD THE ERROR COVARIANCE ESTIMATE
C     P(k+1/k) = (PHI * P(k/k) * PHIT) + Q

      CALL FINDDEL(DEL,DT)
      CALL MATRAN(PHI,PHIT,4,4)
      CALL MATMUL(PHI,PKK,4,4,4,TEMP6)
      CALL MATMUL(TEMP6,PHIT,4,4,4,TEMP7)
      CALL GETQ(XKKM1,DEL,Q)
      CALL MATADD(TEMP7,Q,4,4,1,PKKM1)
      DO 90 I=1,4
      DO 90 J=1,4
          LPKKM1(I,J)=PKKM1(I,J)

90    CONTINUE

C     WRITE(*,*)'P(',TIME,'/',TIMEM1,',0):'
      DO 110 I=1,4
C     WRITE(*,100)(PKKM1(I,J),J=1,4)
100   FORMAT(4F14.4)
110   CONTINUE
120   CONTINUE

      DO 230 L=1,2

C       WRITE(*,*)'****PROCESSING MEASUREMENT # ',L,' ****'
          XDIFF=XKKM1(1,1)-XS(L)
          YDIFF=XKKM1(3,1)-YS(L)
          RANGE=SQRT(XDIFF**2+YDIFF**2)

C UPDATE H MATRIX WITH LATEST STATE ESTIMATES
          H(1,1)=YDIFF/RANGE**2
          H(1,2)=0.0
          H(1,3)=-XDIFF/RANGE**2
          H(1,4)=0.0

C COMPUTE OBSERVATION RESIDUAL
          BRKKM1=ATAN2(XDIFF,YDIFF)
          E(L)=BRG(L)-BRKKM1
```

```
C COMPUTE VARIANCE OF RESIDUALS SEQUENCE
C AND ADAPTIVE GATE VALUE
C     VAR(E)=H*PKKM1*HT+R
        CALL MATRAN(H,HT,1,4)
        CALL MATMUL(H,PKKM1,1,4,4,TEMP1)
        CALL MATMUL(TEMP1,HT,1,4,1,TEMP2)
        VARE(L)=TEMP2(1,1)+R

        GATE1(L)=1.5*SQRT(VARE(L))

C COMPUTE KALMAN GAIN MATRIX
C     G=PKKM1*HT*(H*PKKM1*HT+R)**(-1)

        CALL MATRAN(H,HT,1,4)
        CALL MATMUL(PKKM1,HT,4,4,1,TEMP3)
C        WRITE(*,*)'PKKM1*HT = '
        DO 130 I=1,4
C          WRITE(*,*)TEMP3(I,1)

130      CONTINUE

        FAC1=1/VARE(L)
        CALL MATSCL(FAC1,TEMP3,4,1,G)
C        WRITE(*,*)'G(k) = '
        DO 140 I=1,4
C          WRITE(*,*)G(I,1)

140      CONTINUE

     IF (L.EQ.1) THEN
        G11=G(1,1)
        G13=G(3,1)
     ELSE
        G21=G(1,1)
        G23=G(3,1)
     ENDIF


C COMPUTE UPDATED ESTIMATE
C     X(k/k) = X(k/k-1) + G * E,  WHERE E=Z(k)-H(k)*X(k/k-1)

        XKK(1,1)=XKKM1(1,1)+(G(1,1)*E(L))
        XKK(2,1)=XKKM1(2,1)+(G(2,1)*E(L))
        XKK(3,1)=XKKM1(3,1)+(G(3,1)*E(L))
        XKK(4,1)=XKKM1(4,1)+(G(4,1)*E(L))
```

```
C        WRITE(*,*)'X(',TIME,'/',TIME,',',L,'):'
        DO 150 I=1,4
C          WRITE(*,*)XKK(I,1)

150      CONTINUE

C COMPUTE UPDATED ERROR COVARIANCE MATRIX
C    P(k/k) = (I - G*H) * P(k/k-1)

        CALL MATMUL(G,H,4,1,4,TEMP4)
C        WRITE(*,*)'G*H ='
        DO 170 I=1,4
C          WRITE(*,160)(TEMP4(I,J),J=1,4)
160        FORMAT(4F14.4)

170      CONTINUE

        CALL MATSUB(IMAT,TEMP4,4,4,TEMP5)
C        WRITE(*,*)'I-G*H ='
        DO 190 I=1,4
C          WRITE(*,180)(TEMP5(I,J),J=1,4)
180        FORMAT(4F14.4)
190      CONTINUE
        CALL MATMUL(TEMP5,PKKM1,4,4,4,PKK)

C        WRITE(*,*)'P(',TIME,'/',TIME,',',L,'):'
        DO 210 I=1,4
C          WRITE(*,200)(PKK(I,J),J=1,4)
200        FORMAT(4F14.4)

210      CONTINUE

C  IF THERE ARE MORE MEASUREMENTS
    IF (L.LT.2) THEN

C      USE UPDATED STATE AND ERROR COVARIANCE
C      ESTIMATES FOR NEXT MEASUREMENT
        DO 220 I=1,4
          DO 220 J=1,4
            PKKM1(I,J)=PKK(I,J)
            XKKM1(I,1)=XKK(I,1)

220      CONTINUE
    ENDIF

230  CONTINUE
```

```
C COMPUTE TRUE TRACKING ERROR
      TRKERR=SQRT((XT-XKK(1,1))**2+(YT-XKK(3,1))**2)


C COMPUTE OBSERVATION ERROR
      OBSERR=SQRT((XT-ZX)**2+(YT-ZY)**2)


C SAVE LATEST RESIDUALS FOR AVERAGING
       E1=E(1)
       E2=E(2)


C COMPUTE THE AVERAGE RESIDUAL OVER THE PAST THREE OBSERVATIONS
      M1=(E1+E1M1+E1M2)/3
      M2=(E2+E2M1+E2M2)/3


C PAST THREE RESIDUALS FOR SENSOR 1 ARE : E1,E1M1,E1M2
C BEARING AVERAGE OF SENSOR 1 = M1
C MANEUVER GATE FOR SENSOR 1 = GATE1(1)

C PAST THREE RESIDUALS FOR SENSOR 2 ARE : E2,E2M1,E2M2
C BEARING AVERAGE OF SENSOR 2 = M2
C MANEUVER GATE FOR SENSOR 2 = GATE1(2)

      E1M2=E1M1
      E2M2=E2M1
      E1M1=E1
      E2M1=E2


C COMPUTE ESTIMATED X-Y POSITION, COURSE, AND SPEED
      XPOS=XKK(1,1)
      YPOS=XKK(3,1)

      IF (XKK(2,1).EQ.0 .AND. XKK(4,1).EQ.0) THEN
         HDG = 0.0
      ELSE
         HDG=RTOD*ATAN2(XKK(2,1),XKK(4,1))
      ENDIF

      IF (HDG.LT.0.0) HDG=HDG+360

      SPD=60*SQRT(XKK(2,1)**2+XKK(4,1)**2)
```

```
C WRITE DATA IN OUTPUT FILES

      WRITE(2,*)TIME,XPOS,YPOS,ZX,ZY
      WRITE(4,*)TIME,TRKERR,OBSERR
      WRITE(5,*)TIME,XPOS,YPOS,HDG,SPD


C COMPARE BEARING ERRORS TO MANEUVER DETECTION GATES

      IF ((ABS(M1).GT.(GATE1(1))).OR.
     *        (ABS(M2).GT.(GATE1(2)))) THEN
         WRITE(*,*)'***MANEUVER DETECTION***'
          CALL REINIT(DT,ZX,ZY,ZXM1,ZYM1,LPKKM1,XKKM1,PKKM1)
          E1M1=0.0
          E1M2=0.0
          E2M1=0.0
          E2M2=0.0
          GOTO 120
      ENDIF

      TIMEM1=TIME

      ZXM1=ZX
      ZYM1=ZY

      GOTO 20

240   CLOSE(UNIT=2)

      CLOSE(UNIT=3)

      CLOSE(UNIT=4)

      CLOSE(UNIT=5)

      STOP

      END
```

```
C*************************************************************
C                         SUBROUTINES
C*************************************************************



     SUBROUTINE FINDPHI(PHI,DT)
C*************************************************************
C    COMPUTES THE VALUES OF THE PHI MATRIX
C*************************************************************
     REAL PHI(4,4),DT

     DO 500 I=1,4
     DO 500 J=1,4
     DO 500 K=1,2
        PHI(I,J)=0.0
500  CONTINUE

C COMPUTE PHI MATRIX
     DO 510 I=1,4
      PHI(I,I)=1.0
510  CONTINUE
     PHI(1,2)=DT
     PHI(3,4)=DT

     RETURN
     END



     SUBROUTINE GETQ(XKKM1,DEL,Q)
C*******************************************************
C    SUBROUTINE TO GET Q MATRIX
C*******************************************************
     REAL XKKM1(4,1),Q(4,4)
     REAL QPR(2,2),DEL(4,2),DELT(2,4)
     REAL VARV,VARTH,VT

C       INTEGER FLAG

     IF ((XKKM1(2,1).EQ.0).OR.(XKKM1(4,1).EQ.O)) THEN
       DO 520 I=1,4
        DO 520 J=1,4
          Q(I,J)=0.0
       GOTO 530
520  CONTINUE
```

```
      ENDIF

C  CALCULATE Q' MATRIX

C  LINEAR ACCELERATION = 0.0005 nm/(hr)^2
      VARV  = 0.001

C  ANGULAR ACCELERATION = 0.001 rad/(hr)^2
      VARTH = 0.001

      VT=SQRT(XKKM1(2,1)**2+XKKM1(4,1)**2)
      QPR(1,1)=(((XKKM1(2,1)/VT)**2)*VARV)+((XKKM1(4,1)**2)*VARTH)
      QPR(2,2)=(((XKKM1(4,1)/VT)**2)*VARV)+((XKKM1(2,1)**2)*VARTH)
      QPR(1,2)=((XKKM1(2,1))*(XKKM1(4,1))/(VT**2))*VARV
     *        -(XKKM1(2,1))*(XKKM1(4,1))*VARTH
      QPR(2,1)=QPR(1,2)

C Q=DEL(K)*Q'(K)*DELT(K)
      CALL MATRAN(DEL,DELT,4,2)
      CALL MATMUL(DEL,QPR,4,2,2,TEMP10)
      CALL MATMUL(TEMP10,DELT,4,2,4,Q)
C       CALL MATSCL(0.01,Q,4,4,Q)

530   RETURN

      END




      SUBROUTINE FINDDEL(DEL,DT)
C************************************************
C   COMPUTE THE VALUES OF THE DEL MATRIX
C************************************************
      REAL DEL(4,2),DT

      DEL(1,1)=DT**2./2.
      DEL(1,2)=0
      DEL(2,1)=DT
      DEL(2,2)=0
      DEL(3,1)=0
      DEL(3,2)=DT**2./2.
      DEL(4,1)=0
      DEL(4,2)=DT

      RETURN
      END
```

```
      SUBROUTINE INIT(XS1,YS1,XS2,YS2,BRG1,BRG2,XKK,PKK)
C************************************************************
C     THIS ROUTINE INITIALIZE THE STATE AND ERROR
C     COVARIANCE ESTIMATES
C************************************************************
      REAL XKK(4,1) , PKK(4,4)
      REAL XS1,YS1,XS2,YS2,BRG1,BRG2
      REAL NUMER,DENOM

C INITIAL STATE ESTIMATE

      NUMER=(-YS2*TAN(BRG2))+(YS1*TAN(BRG1))+XS2-XS1
      DENOM=TAN(BRG1)-TAN(BRG2)

      XKK(3,1)=NUMER/DENOM
      XKK(2,1)=0.0
      XKK(1,1)=(XKK(3,1)-YS1)*TAN(BRG1)+XS1
      XKK(4,1)=0.0

C INITIAL ERROR COVARIANCE ESTIMATE

      PKK(1,1)=10000
      PKK(1,2)=0.0
      PKK(1,3)=0.0
      PKK(1,4)=0.0
      PKK(2,1)=0.0
      PKK(2,2)=0.2500
      PKK(2,3)=0.0
      PKK(2,4)=0.0
      PKK(3,1)=0.0
      PKK(3,2)=0.0
      PKK(3,3)=10000
      PKK(3,4)=0.0
      PKK(4,1)=0.0
      PKK(4,2)=0.0
      PKK(4,3)=0.0
      PKK(4,4)=0.2500

      RETURN

      END
```

```
      SUBROUTINE REINIT(DT,ZX,ZY,ZXM1,ZYM1,LPKKM1,XKKM1,PKKM1)
C*************************************************************
C     THIS ROUTINE RE-INITIALIZES THE STATE AND ERROR
C     COVARIANCE ESTIMATES
C*************************************************************
      REAL DT,XKKM1(4,1),PKKM1(4,4)
      REAL ZX,ZY,ZXM1,ZYM1,LPKKM1(4,4)

      XDIFF=ZX-ZXM1
      YDIFF=ZY-ZYM1

      XKKM1(1,1)=ZX
      XKKM1(2,1)=XDIFF/DT
      XKKM1(3,1)=ZY
      XKKM1(4,1)=YDIFF/DT

C     WRITE(3,*)'REINITIALIZED STATES ARE : '
      DO 540 I=1,4
C        WRITE(3,*)XKKM1(I,1)
540   CONTINUE

      PKKM1(1,1)=2.25*LPKKM1(1,1)
      PKKM1(1,2)=0.0
      PKKM1(1,3)=2.25*LPKKM1(1,3)
      PKKM1(1,4)=0.0
      PKKM1(2,1)=0.0
      PKKM1(2,2)=0.1111
      PKKM1(2,3)=0.0
      PKKM1(2,4)=0.0
      PKKM1(3,1)=2.25*LPKKM1(3,1)
      PKKM1(3,2)=0.0
      PKKM1(3,3)=2.25*LPKKM1(3,3)
      PKKM1(3,4)=0.0
      PKKM1(4,1)=0.0
      PKKM1(4,2)=0.0
      PKKM1(4,3)=0.0
      PKKM1(4,4)=0.1111

      RETURN

      END
```

```
      SUBROUTINE MP(XS1,YS1,XS2,YS2,BRG1,BRG2,ZX,ZY)
C*************************************************************
C     THIS ROUTINE COMPUTES THE ESTIMATED
C     X,Y POSITION OBTAINED FROM MEASUREMENTS
C*************************************************************
      REAL ZX,ZY
      REAL XS1,YS1,XS2,YS2,BRG1,BRG2
      REAL NUMER,DENOM

C INITIAL STATE ESTIMATE

      NUMER=(-YS2*TAN(BRG2))+(YS1*TAN(BRG1))+XS2-XS1
      DENOM=TAN(BRG1)-TAN(BRG2)

      ZY=NUMER/DENOM
      ZX=(ZY-YS1)*TAN(BRG1)+XS1

      RETURN
      END




      SUBROUTINE MATMUL(A,B,L,M,N,C)
C*************************************************************
C     THIS ROUTINE MULTIPLIES TWO MATRICES TOGETHER
C     C(L,N) = A(L,M) * B(M,N)
C*************************************************************
C     DIMENSIONS AND DECLARATIONS
      REAL A(L,M),B(M,N),C(L,N)

      DO 570 I=1,L
      DO 570 J=1,N
        C(I,J)=0.0
570   CONTINUE

          DO 580 I=1,L
          DO 580 J=1,N
          DO 580 K=1,M
      C(I,J) = C(I,J) + A(I,K)*B(K,J)
580   CONTINUE

      RETURN
      END
```

```
      SUBROUTINE MATRAN(A,B,N,M)
C************************************************************
C     THIS ROUTINE TRANSPOSES A MATRIX
C     B(M,N) = A'(N,M)
C************************************************************
C     DIMENSIONS AND DECLARATIONS
      REAL A(N,M), B(M,N)
      DO 590 I=1,N
      DO 590 J=1,M
        B(J,I) = A(I,J)
590   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATSCL(Q,A,N,M,C)
C************************************************************
C     THIS ROUTINE MULTIPLIES A MATRIX WITH A SCALAR
C     C(N,M) = Q * A(N,M)
C************************************************************
C     DIMENSIONS AND DECLARATIONS
      REAL A(N,M), C(N,M)

      DO 600 I=1,N
      DO 600 J=1,M
        C(I,J) = Q*A(I,J)
600   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATADD(A,B,N,M,L,C)
C*******************************************
C     THIS ROUTINE ADDS TWO MATRICES
C     { C(N,M) = A(N,M) + B(N,M) }
C*******************************************
C     DIMENSIONS AND DECLARATIONS
      REAL*4  A(N,M),B(N,M),C(N,M,L)
      DO 610 I = 1,N
      DO 610 J = 1,M
       C(I,J,L)=A(I,J)+B(I,J)
610   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATSUB(A,B,N,M,C)
C**********************************************************
C     THIS ROUTINE SUBTRACTS TWO MATRICES
C     C(N,M) = A(N,M) - B(N,M)
C**********************************************************
C     DIMENSIONS AND DECLARATIONS
      REAL A(N,M),B(N,M),C(N,M)

      DO 620 I=1,N
      DO 620 J=1,M
        C(I,J)=A(I,J)-B(I,J)
620   CONTINUE

      RETURN

      END
```

APPENDIX C. FUNCTION PLOT

```
% Function PLOT
%
% This function will plot the results obtained from the program
% SHIPTRACK.FOR which compute the estimation process based on the
% Extended Kalman Filter equations
%
load TRKDATA.DAT
load ERRDATA.DAT
load OUTDATA.DAT
%
% Plotting the Tracking and Observed errors
%
%axis([0 max(ERRDATA(:,1)) 0 max(ERRDATA(:,2))+0.1])
plot(ERRDATA(:,1),ERRDATA(:,2),'-',...
ERRDATA(:,1),ERRDATA(:,3),'-.')
xlabel('TIME (minutes)')
ylabel('ERROR (nautical miles)')
title('(TRACKING ERROR (-)),(OBSERVED ERROR (-.)) vs t.')
grid
pause
meta RESULTS
%
% Plotting the tracking process
%
axis([0 25 0 25]);
plot(TRKDATA(:,2),TRKDATA(:,3),'-',...
TRKDATA(:,4),TRKDATA(:,5),'+g',...
TRKDATA(:,7),TRKDATA(:,8),'+r',...
OUTDATA(:,4),OUTDATA(:,5),'+b',...
OUTDATA(:,2),OUTDATA(:,3),'ow')
gtext('Patrol Boat 1'),pause
gtext('Patrol Boat 2'),pause
gtext('TRUE TRAJECT.  : - ')
gtext('OBS.POSITIONS  : + ')
gtext('EST. POSITIONS : o ')
xlabel('X coordinate (Nautical Miles)')
ylabel('Y coordinate (Nautical Miles)')
title('TRACKING PROCESS')
grid
pause
meta RESULTS
```

```
%
% Plotting the scenario
axis([0 25 0 25]);
plot(TRKDATA(:,2),TRKDATA(:,3),'o',...
TRKDATA(:,4),TRKDATA(:,5),'+g',...
TRKDATA(:,7),TRKDATA(:,8),'+r')
xlabel('X coordinate (Nautical Miles)')
ylabel('Y coordinate (Nautical Miles)')
title('SCENARIO Nr. __ ')
gtext('Patrol Boat 1'),pause
gtext('Patrol Boat 2'),pause
gtext('Start'),pause
gtext('End'),pause
grid
pause
meta RESULTS
```

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